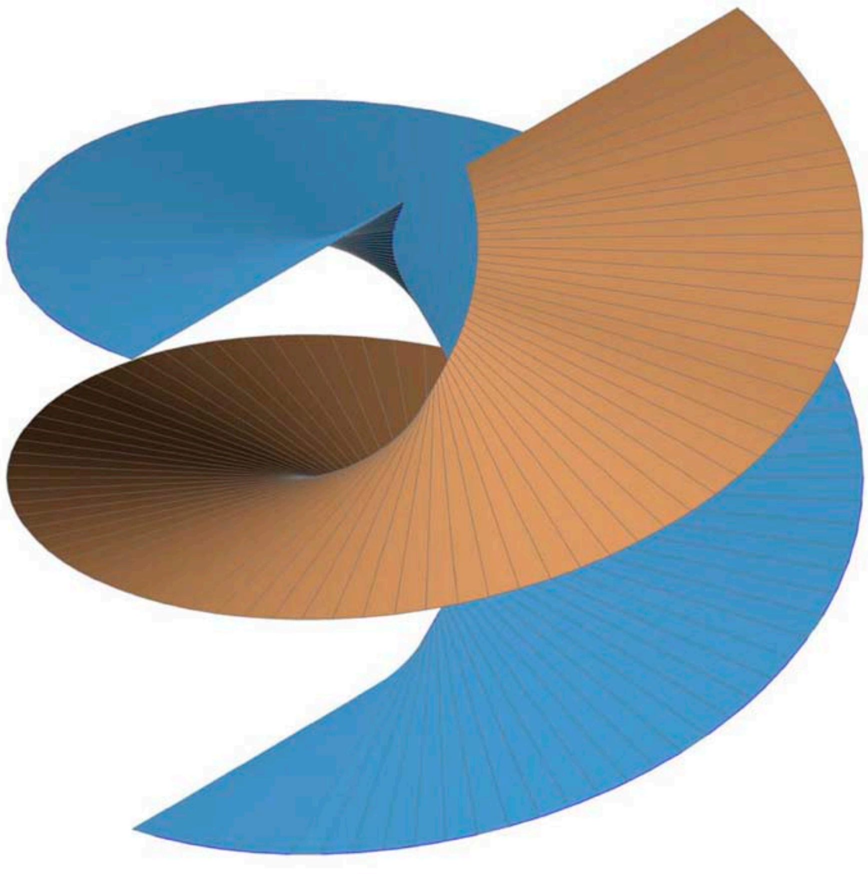
THE DOUBLE HELIX IN ARCHITECTURE AND ENGINEERING



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1. INTRODUCTION – THE SYDNEY OPERA HOUSE CAR PARK

The Sydney Opera House is widely accepted as one of the great buildings, possibly the greatest, of the 20th century. For more than a decade after its completion in 1973 one of the great difficulties for the 2000 or more people attending performances, was the absence of parking within reasonable distance of Bennelong Point. After various false starts over more than 15 years, the New South Wales Government put out a tender in early 1990 for private enterprise to build and operate an 1100 vehicle underground parking station. The facility was required to be built in a restricted footprint area beneath the Botanic Gardens, only a few hundred metres from the Opera House forecourt.

I was part of a team that submitted a design on behalf of the Mulpha Group that comprised two side-by-side rectangular underground structures with cross connections at the ends. Each rectangular structure was much like a traditional above-ground parking station, with narrow ramps and tight corners. And if you were the last patron in either structure you would have to wind your way down seven or eight levels and occupy the last place in the bottom corner. The design had other unattractive features. Ventilation of the two chambers was difficult and expensive, and egress stairs knocked out a significant number of valuable parking spaces. Much to the surprise of some of us, Mulpha's submission was accepted by the Government. Basically the competitors' designs were much the same and Mulpha had a better financial deal.

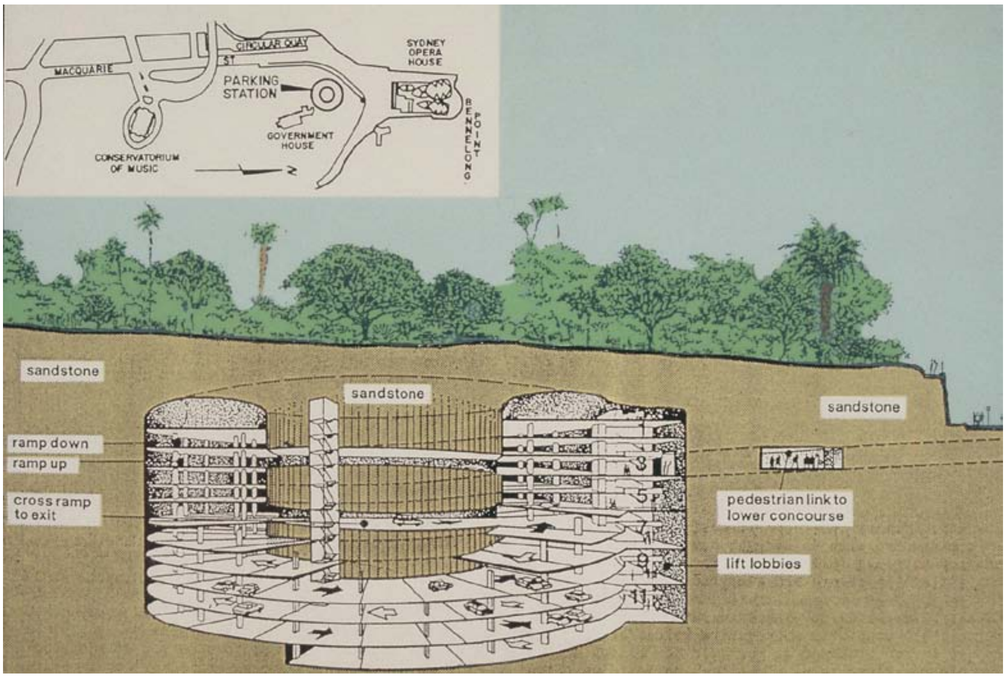
The award was made on a Friday. I went home.

However, one of the projects structural engineers, Warwick Colefax, and the project architect, Ron Barrelle, got to drinking a few beers. And as they mulled over the issues of ventilation, lost parking places due to fire escapes, and having to drive down tight corners into the bowels of the earth to get to the last parking place, Ron Barrelle had an idea. He said he had seen an above-ground parking station in Paris that was a helix, and it self-ventilated up through the spiral. Warwick patiently pointed out that this would not work underground. But then, probably encouraged by the contents and shape of a beer can, the lights started going on and they started to sketch. Within a few hours they had developed the outline of a concept of a 30m deep cavern, circular in plan with a central rock pillar, and this donut shaped cavern contained a double helix reinforced concrete floor structure that could be interconnected at any location simply by tunnelling horizontally through the central pillar of rock. The gently sloping ramps of the helix provided for parking and access. Now the last parking place was not at the bottom, but at the top. Ventilation was easy; only one cavern. Travel distances were minimised and hence the required number of fire stairs was also reduced. The total volume of excavation was reduced, and lo and behold, it could be fitted within the originally allocated development boundaries. The footprint area was reduced from 7900 square metres to 3000 square metres.

A stringent condition of tender was that under no circumstances was the ground surface in the Botanic Gardens to be disturbed for any purpose so the only question was could such a cavern be built, with only 6m of rock cover?

On the Monday I turned up at work, oblivious of Friday's beer befuddled discussion, and the detailed work that followed over the weekend. Mid-morning I received a fax and a phone call. Simple question; could such a cavern be built? I had never seen anything like it. Nobody had, because it had never been done before.

If one is very fortunate then maybe once in a lifetime one will see a concept or design of sheer brilliance. That's what I saw that morning. And we concluded that the cavern could be built with no disturbances to the ground surface.



Sketch of the Sydney Opera House Carpark

By the end of that week the NSW Government agreed that the drawings upon which they had awarded the project could be thrown away, and the double helix was adopted. A year later the facility was opened and it works brilliantly. And that's when my fascination with the double helix began.

Part of the near-completed excavation for the Sydney Opera House carpark; the central core pillar is on the right, the slot in the left hand side is for the stairwell

Who first thought of using a double helix in a building of any sort? And then who first thought of using a single helix, of making a screw?

My quest has taken me to interesting places, and to thought-meetings with fascinating people, from Archimedes to an Iranian architect in 12th century Afghanistan. And this journey I would like to share with you.

2. THE CONTINUOUS HELIX

A spiral is a curve that winds around a fixed point with continuously increasing radius, like a flat-coiled rope.

A helix is an inclined plane wrapped around a cylinder. It is quite a sophisticated three-dimensional shape, but appears in nature in vines climbing up a tree, and within some seashells. As such the idea of a helix would have been apparent to early humans. However, the question as to when did a human being first make use of a helix in a tool or in a building is fascinating and difficult to answer.

As a school boy I, like so many, had been intrigued by the story of how Archimedes solved the problem of whether or not Hieron, the King of Syracuse, had been diddled in regard to the gold content of a new wreath. In effect Archimedes worked out how to determine the volume of this complex piece of jewellery, and in the process developed the foundation of the science of hydrostatics, the behaviour of fluids at rest. At school I also learned that at the time of the ancient Egyptians, irrigation from the Nile had been hugely improved by replacement of the old shaduf, a bucket on the end of a seesaw arrangement, by a device called an Archimedean screw. Little was I to know that 45 years later my search for the first recorded application of a helix engineering would lead me back to this Egyptian "pump".

It is not possible to say who has been the greatest genius: Beethoven, Einstein, Newton, Leonardo da Vinci, Michelangelo? It is not even possible to say who has been the greatest mathematician: Gauss, Euler, Newton? However, it is of little doubt that Archimedes was the greatest mathematician and, probably genius, of antiquity. What he described as The Method is, post-Newton, called Integration. He used this approach to work out that the ratio of a circumference to a circle to its diameter, π, lies between 220/70 and 220/71. He arrived at these upper and lower bounds by starting with one hexagon that circumscribed a circle, and one that inscribed the circle, and then successively increased the number of sides until the hexagons were 96 sided figures that trapped the circumference of the circle.

Archimedes was a Sicilian and was born in about 287 BC. As a young man he went to Alexandria to study mathematics and there either overlapped with, or followed just after, Euclid and his fellow mathematicians who were responsible for producing the most important mathematical textbook for all time, "Elements". The Museum of Alexandrian was at that time a type of independent research institute funded by the Ptolomies, who followed on from the Egyptian pharaohs.

There is no doubt that Archimedes discovered the helical water screw while at Alexandria. Whether he invented it is another matter, and one discussed eruditely by Dalley and Oleson[1]. This device is ascribed to Archimedes by certain ancient writers such as Diodorus of Sicily (about 50BC) in relation to irrigation from the Nile, and particularly, in a wonderful description of Roman mine dewatering in Spain:

> "At a depth they sometimes break in on rivers flowing beneath the surface whose strength they overcome by diverting their welling tributaries off to the side in channels. Since they are driven by the well-founded anticipation of gain, they carry out their enterprises to the end, and most incredibly of all, they draw off the streams of water with the so-called Egyptian screw, which

[1] Stephanie Dalley and John Peter Oleson. "Sennacherib, Archimedes, and the Water Screw", Technology and Culture, Vol 44, Jan 2003.

Archimedes the Syracusan invented (or 'thought about', or 'observed'[2]) when he visited Egypt".

Vitruvius (1st Century BC) provides a description drawing of an eight helix water screw, but does not ascribe it to Archimedes.

Vitruvius' description is as follows.

"1. There is a machine, on the principle of the screw, which raises water with considerable power, but not so high as the wheel. It is contrived as follows. A beam is procured whose thickness, in digits, is equal to its length in feet; this is rounded. Its ends, circular, are then divided by compasses, on their circumference, into four or eight parts, by diameters drawn thereon. These lines must be so drawn, that when the beam is placed in an horizontal direction, they may respectively and horizontally correspond with each other. The whole length of the beam must be divided into spaces equal to one eighth part of the circumference thereof. Thus the circular and longitudinal divisions will be equal, and the latter intersecting lines drawn from one end to the other, will be marked by points.

2. These lines being accurately drawn, a small flexible ruler of willow or withy, smeared with liquid pitch, is attached at the first point of intersection, and made to pass obliquely through the remaining intersections of the longitudinal and circular divisions; whence progressing and winding through each point of intersection it arrives and stops in the same line from which it started, receding from the first to the eighth point, to which it was at first attached. In this manner, as it progresses through the eight points of the circumference, so it proceeds to the eighth point lengthwise. Thus, also, fastening similar rules obliquely through the circumferential and longitudinal intersections, they will form eight channels round the shaft, in the form of a screw.

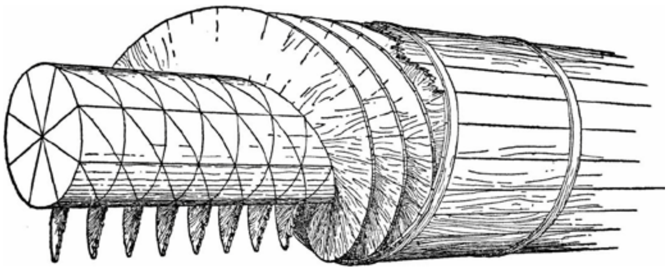
3. To these rules or slips others are attached, also smeared with liquid pitch, and of these still others, till the thickness of the whole be equal to one eighth part of the length. On the slips or rules planks are fastened all round, saturated with pitch, and bound with iron hoops, that the water may not injure them. The ends of the shaft are also strengthened with iron nails and hoops, and have iron pivots inserted into them. On the right and left of the screw are beams, with a cross piece at top and bottom, each of which is provided with an iron gudgeon, for the pivots of the shaft to turn in, and then, by the treading of men, the screw is made to revolve.

4. The inclination at which the screw is to be worked, is equal to that of the right angled triangle of Pythagoras: that is, if the length be divided into five parts, three of these will give the height that the head is to be raised; thus four parts will be the perpendicular to the lower mouth. The method of constructing it may be seen in the diagram at end of the book. I have now described, as accurately as possible, the engines which are made of wood, for raising water, the manner of constructing them, and the powers that are applied to put them in motion, together with the great advantages to be derived from the use of them."

From this description someone produced a sketch (reproduced below) which appears in many publications, ascribed to Vitruvius. However, none of Vitruvius original

[2] Alternative translations of "∊νρ∊ν"

illustrations survive and all those in the 55 known handwritten manuscripts (earliest 1486) of *De Architectura*[3] are interpretations from the period of the given edition.



Eight helical screw pump as interpreted from Vitruvius' description

It has been suggested that the invention of the helical pump, or water screw, was an outworking of Archimedes' mathematical studies, in particular his work "On Spirals". However, the 28 propositions in "On Spirals" deal only with the two-dimensional figure that is the definition of a spiral. Nowhere in that book, or in his two books on hydrostatics ("On Floating Bodies") does he mention helices or the water screw.

Dr Stephanie Dalley, a specialist in Assyrian archaeology at Oxford University, provides what to me is compelling evidence that my hero, Archimedes, did not invert the water screw. She points to Sennacherib who ruled Assyria for 24 years, some 400 years before Archimedes. At the height of his powers Sennacherib had conquered the Chaldeans to the south (including the city of Babylon on the Euphrates River), the Phoenicians and Israelites to the west and probably a fair piece of Egypt. His hometown was Nineveh, on the Tigris River, north west of modern-day Bagdad. By about 700 BC he ruled a vast area that stretched from Southern Turkey to the eastern border of Egypt, and from the Caspian Sea to the Arabian Gulf. Nineveh was rebuilt with magnificent palaces, a city wall, an administrative centre, parks, and above all an immense irrigation system. He built a canal for 48km from a dam on the River Gomel to the north. He built another dam to the east to control the river Khasr that flowed through Nineveh. Stephanie Dalley notes that in all he built 18 different canal systems including tunnels and aqueducts. "Provided that the technology was available, he had the manpower and the raw material to achieve whatever the wanted, regardless of time, expense, or detriment to the health of his workmen".[4] However, where matters got really interesting in relation to our quest for the first use of a helix, is Dalley's studies of a cuniform inscription where Sennacherib, in an Akkadian inscription written on clay prisms, describes his ability to make huge hollow copper and bronze castings and then:

[3] Vitruvius served as military engineer in the armies of Caesar and Augustus and worked with Marcus Aurelius, Publius Minidius and Gaius Cornelius to build projectile artillery and other war machines.

Vitruvius began writing his famous work De architectura libri X, which he dedicated to Augustus, even before 33 B.C. and apparently completed it before 22 B.C. Unfortunately this book is said to be full of gaps and many of its passages are unclear and ambiguously formulated.

Many important artists of the Renaissance, including Leonardo da Vinci, studied Vitruvius thoroughly. Vitruvius is one of the first Ancient authors whose work was printed (Editio princeps around 1486) and was translated into many European languages. The numerous sixteenth-century editions are important not only because of their commentaries and translations, but primarily because of the illustrations they contain.

[4] Dalley and Oleson, 2003

"In order to draw water up all day long I had ropes, bronze wires and bronze chains made, and instead of shadoofs I set up the great cylinders and alamittu palms over cisterns. I made the royal lodges look just right. I raised the height of the surroundings of the palace to be a wonder for all peoples. I gave it the name: Incomparable Palace. A park imitating the Amanus mountains I laid out next to it, with all kinds of aromatic plants, orchard fruit trees ……"

Dalley postulates that the use of the words 'alamittu palm' was used as a metaphor to indicate a helix, and that Sennacherib had cast bronze or copper screw pumps.

No remains of such pumps have been found, but it is pretty certain that he had the technology. In the Louvre is a bronze cylinder, from the middle East, cast some 1200 years BC, measuring 4.4m long, 180mm diameter and with a wall thickness of 15mm. In the British Museum is a Relief from the palace of Ashurbanipal, Nineveh that includes palm trees with stem patterns formed by crossing helices and from the same period are Assyrian cylindrical stamps that, when rolled on a clay pad, created pictures including palm trees with helical stems.[5]

Helical palms, detail from top part of a limestone relief from the palace of Ashurbanipal who established a very large library at Nineveh during his reign, 668BC – 627BC

Dalley provides good support for the view, that the famous Hanging Gardens of Babylon were not in Babylon but were Sennacherib's gardens in Nineveh, a place that later became known as Old Babylon. The Hanging Gardens were described by Strabo who, at about the time of Christ, wrote an amazing Geography of the then known world, and by Philo who states:

"streams of water …… are partly forced upwards through bends and spirals to gush out higher up, being pushed through the twists of these devices by

[5] The Illustrated Bible Dictionary, Part 3, p1621.

mechanical forces. So, brought together …… these waters irrigate the whole garden".

These descriptions of the Hanging Gardens post-date Sennacherib by 600 years and don't prove that there were water screws at Nineveh, but they do support the hypothesis.

On balance it rings as highly probable that before Sennacherib's time someone invented a water-lifting device using some form of helix made from wood, or maybe the natural hollow helix of the horns of a Mountain Goat. In Sennacherib's time a device of this nature was cast in bronze and/or copper. The Assyrians would then have introduced the concept to the Nile Valley in about 700 BC where it was the perfect device for lifting mud-laden waters, and where it was probably manufactured from wood. And, probably, that's where Archimedes first saw the device and subsequently took it to Europe. But then – maybe not. Possibly the young Archimedes had an eureka moment in Alexandria, long before as an older man he floated in the public baths in Syracuse and suddenly understood hydrostatics. But I don't think so, my vote is that the first use of a helix in a machine was in a water screw, somewhere in the area of Iran and Iraq, alongside the Tigris or Euphrates rivers, and more than 700 years BC.

But was this the first use of a helix in engineering? Was it used earlier in buildings?

A modern Archimedes Screw Pump, Kinderdijk, photo M A Wijngaarden

3. THE FIRST HELIX IN ARCHITECTURE

We do not know, and will never know, when homo sapiens first thought of using a single helix to make a tool or build a structure. In a remarkable, if somewhat long-winded, book published in 1915, Sir Theodore Cook[6] presents "An account of spiral formations and their application to growth in nature to science and to art". In this book he covers a wide range of natural spirals and helices but does not deal with the civilisation of Mesopotamia. He notes that the idea of a helix could have been gained from the human umbilical cord, from the horn of a Kudu, or from many shells and plants.

A near perfect natural helix created by a vine in the jungle on the island of Borneo, Indonesia

Cut-away Voluta shell, Pacific Ocean, NSW

Fasciolaria shell, Pacific Ocean, NSW

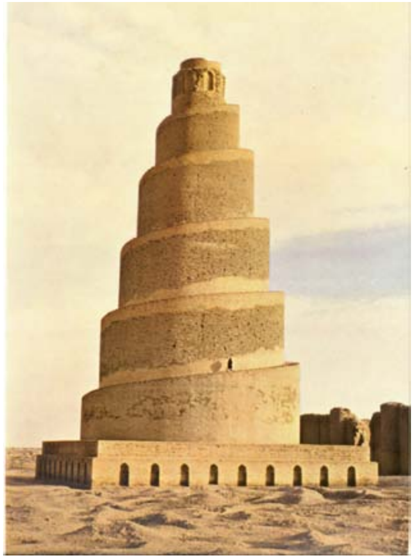
Voluta shell, Pacific Ocean, NSW

Personally I imagine that some 20000 years ago, or thereabouts, a Cro Magnon playing with the shell of a *Turritella Duplicata*, or a *Pleurotama Monterosatoi*, or any other of the numerous helical shells, discovered that when he or she twisted it into a layer of clay it "screwed" itself in, and resisted being pulled out. Certainly we know that such shells were strung together in necklaces in Cro Magnon times.

Cro Magnon's no doubt made simple steps in and out of their cave shelters, if by no other means than by rearranging lumps of limestone, but we know they didn't make spiral staircases. The first spiral stairways were probably the inclined planes constructed around tower structures built by the Babylonians in about 6000 BC. Most of the early structures of this type, exposed by archaeological digs, are gently

[6] Reprinted, Dover Publications, New York, 1979.

inclined ramps, with no steps. Certainly by the 800s AD they had helical ramps. However, given the obvious nature of a step it is easy to believe that the Babylonians did have simple stepped helical ramps winding up the outside of their squat towers. So to answer the question posed in the previous chapter, it is probable that the first use made by homo sapiens of a helix in architecture and engineering was around a tower in Babylonia. However, these were very unsophisticated structures and to find one of the first true helical staircases we have to travel in our minds to Egypt, not to the pyramids of 2500 BC, but to the later times of the early Greeks.



Minaret of the Friday mosque, Samarra, built ~850 AD

From about 300 BC there stood at Alexandria a lighthouse built from white marble. It is said to have been about 110 metres high (a 35 story building) and was finished during the reign of Ptolemy II, in about 280 BC. According to ancient Greek texts the lighthouse was built in three stages, all sloping slightly inwards from the base. And inside was our point of interest, a spiral staircase from the base to the top, where a fire burned at night. Apparently the lighthouse collapsed in an earthquake in the 1300's. In AD 1477 the Mamluk Sultan Qa'it Bay built a fort from its ruins.

Archimedes was in Alexandria in about 250 BC and maybe he climbed up the lighthouse to keep fit. Certainly he would have returned to Syracruse with two uses of helices in his head, the screw sump and the spiral staircase.

Interestingly it is also in Alexandria we must make our next stop, for a predecessor of a Leonardo da Vinci masterpiece that we will meet when, in due course, we move on to Orvieto in Italy.

Carved out of rock near what was the fishing port of Rhakotis, the oldest part of Alexandria that predates Alexander the Great, is a "tour-de-force of rock architecture". This is Kom el-Shuqafa, or the "Mound of Shards". It is an underground burial place, also known as the Great Catacomb. It was carved out of limestone in about 100 AD. The complex consists of a ground level vestibule and then a deep spiral stairway that is wrapped around a 6m diameter shaft. The shaft acted as a light well and is separated from the helical staircase by a wall consisting of squared blocks pierced by arched windows that have slanted sills in order to direct sunlight onto the stairs. There are 99 steps that decease in height as they approach the surface. It is suggested that this design was to make it easier for the tomb visitors on the long climb out after viewing the deceased in the lower levels.

The Great Catacomb is unique in representing a melding and mixing of Egyptian, Greek and Roman cultures. The funerary motifs are pure ancient Egyptian, but the architects and artists were trained in the Greco-Roman style. The decorations are of ancient Egyptian themes but with a Greco-Roman presentation that makes them unlike anything else in the world.

So we have reached the point in our quest that would suggest that the use of a single helix in a spiral staircase, as opposed to a ramp around a tower, saw the light of day at about the same time as the helical pump, sometime around 1000 to 700 BC.

The available evidence is that these applications predated the use of a simple screw by many hundreds of years.

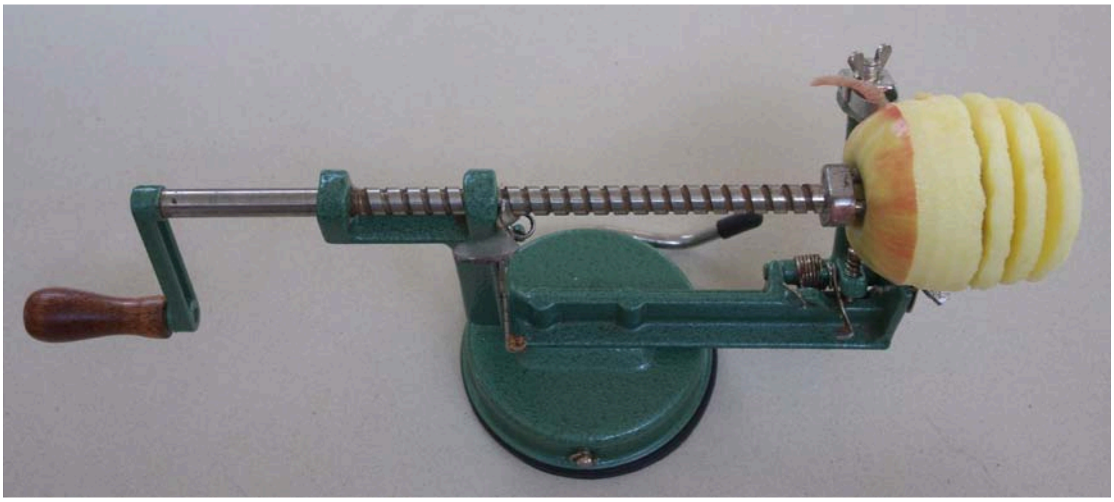
Rybezynski, in his lovely little book, *One Good Turn*[7] being "A natural history of the screwdriver and the screw", accredits Hero of Alexander (10 to 70 AD) as the inventor of the screw press, and Archimedes, through his water pump, as the inventor of "the first appearance in human history of the helix". As already explained, I don't agree with this conclusion, but I do accept Rybezynski's conclusion that the metal screw was first used sometime in the 1400s. I also accept Rybezynski's conclusion that the screw and screwdriver are the most important tools ever invented; or maybe we should say; copied from nature.

Their use permeates our lives, from the tiniest screw, to opening bottles of wine, and to Italian tomato crushers for making Conserva di Pomodora.

Helix for tomato crushing

Everyday helices

[7] Rybezynski, W. One Good Turn, Soribner, 2000.

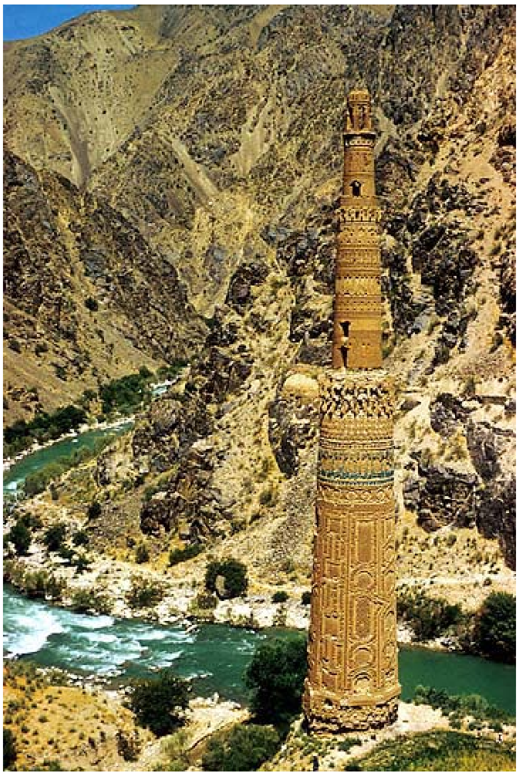


A helix used to core and peel an apple and slice it into an apple helix

However, the trigger for my quest was the beautiful simplicity of the double helix as used in the Sydney Opera House underground parking station. And we must continue our mental journey to find when the elegant simplicity of the double helix was first invoked, and how it was used in other structures before finding a home in the sandstone bedrock next door to Utzon's masterpiece.

Natural double helix; rainforest, Mosman Gorge, Australia

This search has taken me from the entry vestibule of the Vatican Museum, into the chateau's of the Loire Valley, and down a 15th Century well in Orvieto. It ended in the most unexpected place, in a Qantas aeroplane, 35000 feet above the Simpson Desert, when I saw a single picture of a structure in the badlands of Afghanistan, a structure so remarkable that if it is not the world's first double helix in a building it deserves to be.



The Minaret of Jam, Afghanistan

The Minaret of Jam, the world's second tallest minaret, rises in solitude from the floor of a narrow, rugged, barren valley in Ghur Province, Afghanistan. In bland terms this is a 60.4m high tower, built in 1175 containing a double helix stairway. But bland this structure is not, and before presenting its detailed technicalities and historical context, its real flavour can be appreciated from the following article by Rory Stewart in the August 2002 New York Times magazine.

> *"In a deserted maze of narrow gorges in the central mountains of Afghanistan, I turned a corner and saw a tower. It rose 200 feet, a slim column of intricately carved terra cotta set with a line of turquoise tiles. There was nothing else. The mountain walls formed a tight circle around it, and at its base two rivers, descending from high mountain passes, ran through the ravines into wilderness.*
>
> *I was crossing Afghanistan on foot, and it had taken me two weeks to walk to this spot from Harat, the principal city of western Afghanistan. The valley of Jam as the area is called, was a place of relative tranquillity, protected by high mountains from the pro-Taliban feudal lord to the south and the anti-Taliban feudal lord to the north. There was no human in sight, no sound, no sign of the last 24 years of Afghan war. There was only a tower of pale, slender bricks, more than 800 years old according to an inscription at the top of the tower. A dense chain of pentagons, hexagons and diamonds wound round the column. And in Persian blue tiles, the colour of an Afghan winter sky, on the neck of the tower, the words: "Ghiyath al-Din Muhammad, King of Kings …."*
>
> *The tower of Jam was first visited by a foreigner in 1957. Several archaeologists subsequently made the difficult journey, but they were unable*

to decide what the tower had been. The Russian invasion of 1979 stopped further visits from Western scholars. Some archaeologists concluded that it had been part of a mosque, called it the minaret of Jam and looked for the lost city of the Turquoise Mountain in the valley. They discovered very little except, to add to the mystery, a small 12th-century Jewish cemetery a mile and a half from its base. Others asserted that this was a pre-Muslim holy site and that the tower had been built to mark the arrival of Islam in this most lonely and sacred spot. Whatever their differences, the archaeologists had managed to agree on two things – that the tower was a uniquely important piece of early Islamic architecture and that it was in imminent danger of falling down.

In the last decade, most of Afghanistan's cultural heritage was removed or destroyed; the Kabul Museum was looted and the Bamiyan Buddhas were dynamited by the Taliban. By the time of my visit, officers of the Society for the Preservation of Afghanistan's Cultural Heritage had received no reliable reports on the tower of Jam for six months. Indeed, no one in Kabul was sure whether the tower was still standing.

I went inside the tower and began climbing the steep steps. With considerable difficulty, I managed to ascend perhaps 120 feet, emerging into a circular chamber. I continued up, climbing between portions of an old staircase, until I emerged just below the lantern, where the muezzins would step out to sing the call to prayer. Above me were smoke-blackened wooden beams, which must have once supported an external balcony. I looked out from the skylight and saw on the facing ridge two small ruined towers and, to my surprise, a line of trenches cut into the gravel slope.

When I emerged from the tower, I found a man squatting on the ground, stroking his long beard. Standing to greet me, he said in Persian: "Peace be upon you, may you not be tired. How are you? I hope you are well?" and other politenesses, all at a rapid pace with no pause for an answer.

I gathered that this man was Bushire, the legendary fighter who was said to have led 80 men against the Soviets and then, during the last five years, fought the Taliban. I had a letter of recommendation to him in Persian and took it out, but he waved it away because it was very likely he was illiterate. Instead, he invited me to his house.

Below, near his mud house, I noticed a curious stone lying on the ground. I picked it up and found that it was a piece of gray marble carved with a floral frieze. Inside the guest room, we sat on the carpets while Bushire's son fed the fierce fire in the stove with dry twigs.

"What are you doing at the moment?" I asked.

"I am a director of a society which has been set up to protect the tower," Bushire said. "We get money from foreigners abroad to preserve its history."

"And have you found out anything about the history of the tower?"

"Well, we've dug up quite a lot of stuff from the ground."

"What kind of things?"

“Oh, we have sold most of them to traders from Heart, but I’m sure there are a few pieces left. Son, go and see what there is next door.”

His son, Abdullah, returned with a tray of green tea and some objects wrapped in a cloth. There was a marble slab with a floral pattern (like the piece that I’d found outside); a terra-cotta ewer, apparently from the 12th century, covered with a bold black design of waves and fish eyes; a bronze six-sided die with five spots on each side; a hemispherical bead carved from bone; and a large clay disc with a peacock in the center.

“And where are these from?” I asked.

“From all over the mountainside.”

After tea, I climbed up the hill beside the tower. The gravel was loose and the slopes steep, and I needed to use my hands. I soon found myself clambering over rough trenches, some almost 10 feet deep. Along the rim of the pits were piles of sand and broken fragments of pottery. I passed shards of brilliant yellow porcelain, half of a terra-cotta bowl, a section of ancient gutters and some new spades and pick axes. Clearly the antique robbers did not steel one another’s tools.

Those digging had made no attempt to preserve the shape of the buildings they had found; only in a tiny section on the ridge could you even trace the walls of the rooms. The villagers were tunneling as deeply and as quickly as possible to reach whatever lay beneath, and destroying a great deal in the process. The trenches, which had been invisible from the base of the tower, now stretched across every slope in sight. The villagers seemed to have succeeded where the archaeologists had failed. They had uncovered what looked like an ancient city – and where rapidly laying it waste.”

The minaret of Jam is covered with geometric and floral brickwork and turquoise-glazed epigraphic bands. One of the many peculiarities of the minaret is its almost exclusive dependence on varieties of angular script at a time when cursive had been in common use for hundreds of years for monumental inscriptions in that region. The sole use of cursive is for the architect’s name, or signature, one Ali ibn Ibrahim al-Nisaburi. This suggests that Ali, or his family, was from the eastern Iranian city of Nishapur. Now this is where it gets very interesting.

Apart from it’s mines being the world’s source of turquoise (copper aluminium phosphate) for almost 2000 years, and hence the turquoise ceranuis on the minaret, Nishapur was the home of Omar Khaygám. He lived from 1048 to 1131, about 100 years before the minaret of Jam was built, and whilst most of the westerners know of him as a poet, he was a brilliant mathematician. Given that he cracked the difficult problem of solving cubic equations, by a geometrical method of intersecting a circle with a parabola, I think he would have determined the elegance of a double helix, and worked out its simple geometry, whilst eating his cornflakes. But let us return to the minaret.

The mountainous province of Ghur, in which the minaret stands, was a marginal region of the eastern Islamic world that converted to Islam only in the ninth or tenth centuries – much later than many of the surrounding areas. Ghur enjoyed its brief moment of glory in the half century after 1150, when one of the maliks (chiefs) of the region sacked Ghazna, the eponymous capital of the Ghaznavid sultans who had dominated the eastern Islamic world for almost one hundred and fifty years, and assumed the title of sultan. This dramatic event, which earned its perpetrator the sobriquet Jahan-soz (World-burner), marked the abrupt entry of these mountain chiefs onto the wider political stage and begin their meteoric rise from regional obscurity. The apogee of the Ghurid sultanate was reached under the reign of the sultan brothers Ghiyath al-Din Muhammad b. Sam (1163 to 1203) and Mu'izz al-Din Muhammad b. Sam (1173 to 1206). At its zenith in the last decade of the twelfth century, the Ghurid sultanate stretched from the Iranian metropolis of Nishpur in the west to the Indian city of Benares in the east, from the steppe of Central Asia in the north to Sind in the south. The floruit of both dynasty and region was short-lived; the death of Mu'izz al-Din in 1206 effectively marked the end of Ghurid power and the disintegration of the sultanate.

The minaret probably marks the site of the city of Firuzkuh which was the capital of the Ghurid dynasty. The city itself was destroyed by the Mongols in 1222 but fortunately they didn't knock down the minaret.

The tower, at the junction of the *Hari-rud* River and its tributary *Jam-rud*, in the western extension of the *Hindukush* Mountains. The minaret rises on an octagonal base (nominally 9m in diameter) and in this aspect departs from most contemporaneous minarets that have circular bases. The octagonal base transforms into tapering circular sections for the rest of its elevation. There are in effect three tapering cylindrical shafts with platforms at each step change in diameter. The original entrance of the tower, which is currently inaccessible lies below 4 to 6 metres of alluvial deposits from the adjoining river. The inner chamber is ascendable through a pair of helical stairs supported on a central tapering solid shaft. These stairs terminate at the first step at a height of 40.8m. The level of foundation has not yet been ascertained. From existing information, the base of the tower has been considered six metres below the present level of the approach embankment for the structural assessment.

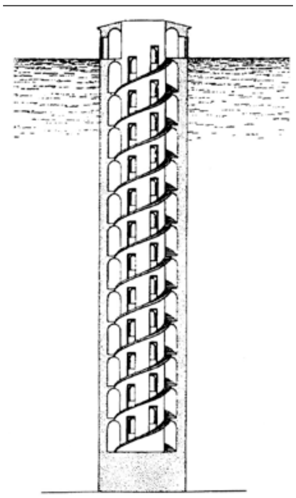
Double helix staircase in the Minaret of Jan, Afghanistan

The tower has an inclination of 3.4° north-north-eastwards, for a reason which is yet to be ascertained, but probably attributable to scouring due to its precarious location at the junction of two rivers. The monument is on UNESCO's World Heritage List and various threats to it have prompted the World Heritage Committee to incorporate it on the List of World Heritage in Danger since June 2002. Preliminary investigations have revealed that one edge of the base section of the tower is very close to being in a state of tensile stress.

4. AN EXTRAORDINARY WELL AND A WONDERFUL STAIRCASE

About 100km north of Rome is the hilltop village of Orvieto. Apart from having an amazing cathedral, and one of the world's greatest cappuccino shops, it has a most extraordinary well, the Pozzo di San Patrizio, carved into the tuff, and with donkey access by a double helix. The story is as follows.

In 1527, following the sacking of Rome, Pope Clemente VII, (Giulio de Medici) retreated to the Albornoz fortress on the steep sided volcanic mesa of Orvieto. In order to provide a safe water supply in the event of a siege, Antonio da Sangallo the Younger (1484-1546) designed a well, 62m deep, to access fresh water in the porous tuffa. The feature that makes this well so special is that carved out of the rock around the circumferences of the well are narrow ramps in a double helix. There are windows from the sides of these ramps into the well. Donkeys, carrying empty buckets could walk down one ramp and not have to pass their loaded mates slogging up into the companion ramp.



1808 drawing of the well of San Patrizio, Orvieto (excavated ~1527)

View down the well of San Patrizio, Orvieto. There is a bridge just above the water at the bottom of the well.

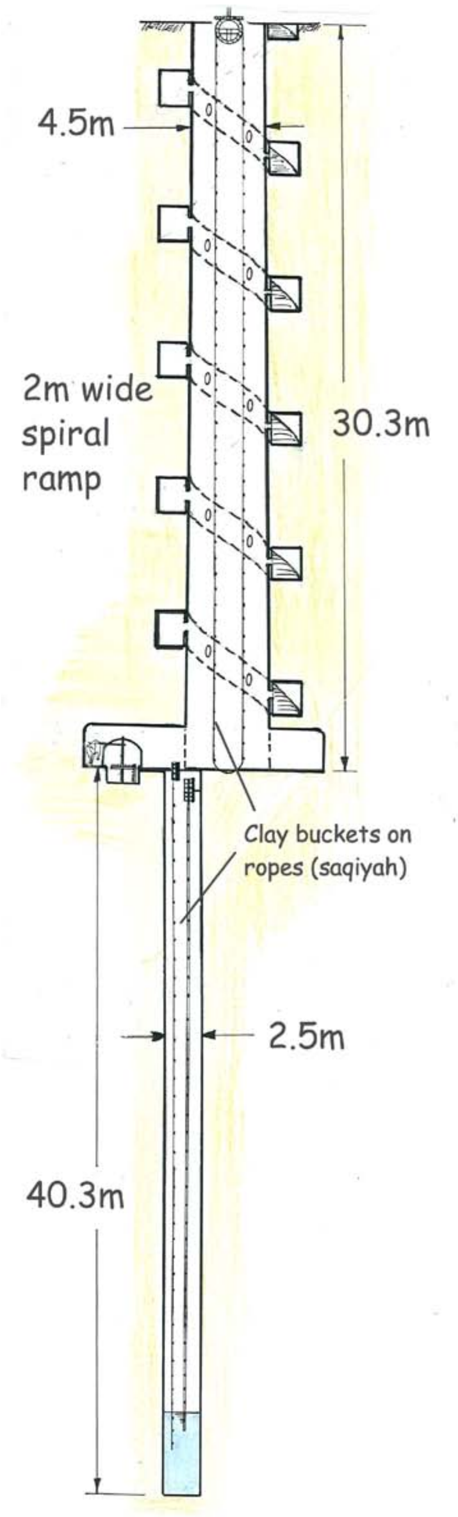
It is reasonable to deduce that the Orvieto well was a development of an equally remarkable well excavated in about 1176 under the direction of Salah al-Din Yusuf ibn Ayyub, known to Westerners as Saladin[8], at the Citadel (Qal'at al Jabal) in Cairo.

Saladin's early priorities were to protect Egypt from further Crusader attacks and to secure his position as Caliph. He set in motion the building of a citadel with a new enclosure on the Maqattam hills overlooking Cairo. One of the first tasks undertaken by his loyal vizier, Qaraqush, was a well to safeguard the water supply. The work was done by Christian prisoners-of-war; it is reported that some 50,000 of such men were used in building the citadel[9] – a number I find difficult to believe.

The well is known as Joseph's (Yusuf's) well, or the Spiral Well, or *Bir al-Halazon*[10]*.* It was documented in detail as part of the work of more than 150 scholars and scientists who accompanied Napoleon's army, the occupiers of the Citadel from 1789

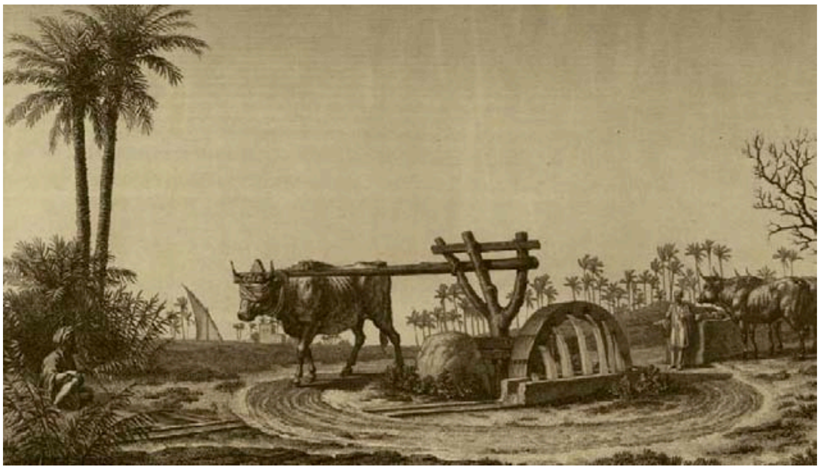
[8] General Saladin was sent from Syria to defend Cairo against the Crusaders. Having succeeded in this he eventually established an empire that extended from Cairo to the borders of Mesopotamia.
[9] Aramco World, March/April 1993
[10] "the Citadel of Cairo", Ed Nasser Rabbat, The Aga Kahn Trust for Culture, 1989

to 1801. These 'savants' produced an extraordinary 20 volume treatise on Egypt, including 11 volumes of beautiful drawings. The following figure is my tracing of the part of Plate 73 of Volume 7 that includes a scale drawing of the well.

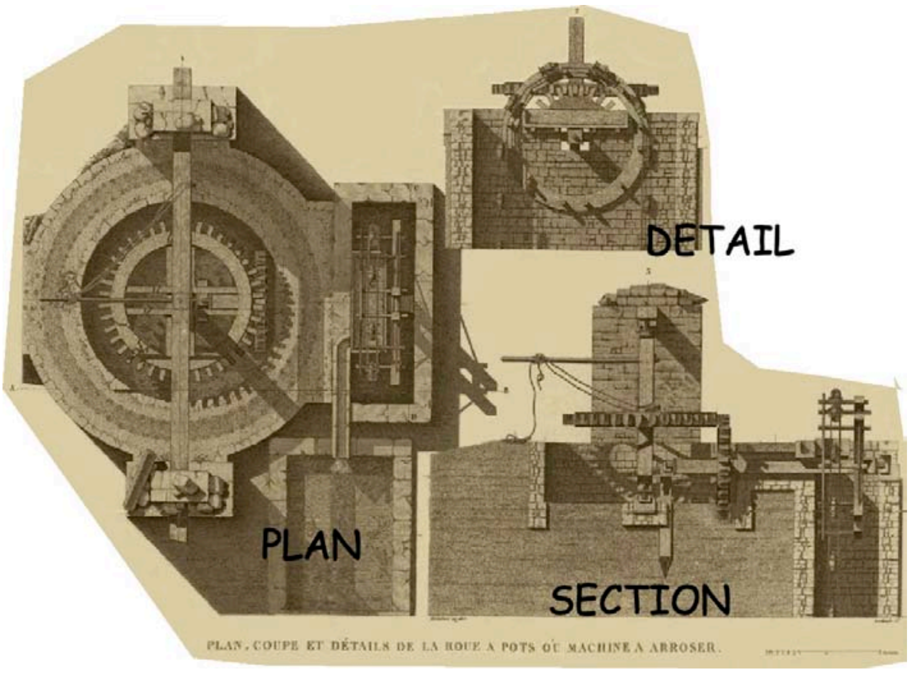


Yusuf's Well, The Citadel, Cairo (*from Vol 7 Recueil des Observations et des Researches, Quiont ete Faites en Egypte. Pendant L'Expedition de L'Armee Francaise, Paris 1822*)

It can be seen that the well comprises two shafts, slightly offset. They are square, the upper 4.6m by 4.6m, and the lower 2.5m by 2.5m. The water was lifted by two *saqiah,* comprising wheels turned by oxen that, in turn, operate 30m and 40m loops of rope carrying clay pots. A 2m wide helical ramp around the upper shaft allowed oxen and operators to reach the top of the first stage of the well[11]. At this level the water was tipped into a cistern from which it was lifted to the surface by the upper *saqiah*.



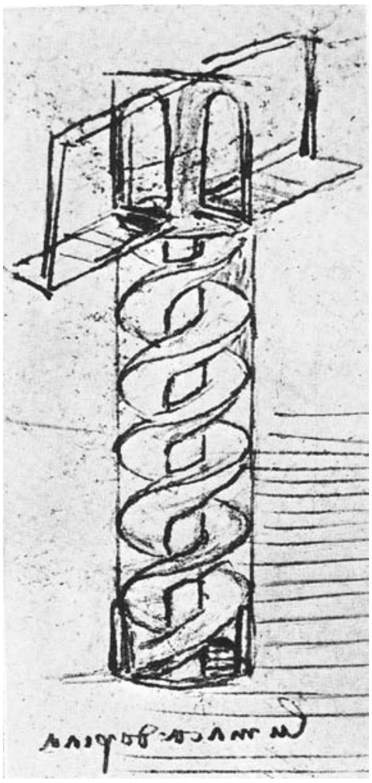
Cattle Operating Water Wheel *(from Vol7 Recueil des Observations et des Recherches, 1822)*



The Mechanics of a *Saqiah* *(from Vol7 Recueil des Observations et des Recherches, 1822)*

[11] The drawing by Napoleon's scientists show the helical ramp terminating at the top of the lower *saqiah* but a drawing in the 1989 publication "The Citadel of Cairo" shows a narrower helical ramp extending to the bottom of the well. This would appear to be reasonable as it would have been nigh unto impossible to excavate a 2.5m by 2.5m, 40m deep, vertical shaft without such spiral access and the ventilation thereby provided.

It would seem reasonable to conclude that the idea for the form of construction of Yusuf's well came from the Kom el-Shuqafa at Alexander, and it is quite possible that Sangallo The Younger knew about both these structures when the Pope asked him to provide a secure water supply at Orvieto. But what Sangallo did that was so clever, was to add the second helix, so that donkeys going down did not have to pass the fully laden donkeys ascending from the bottom of the well.

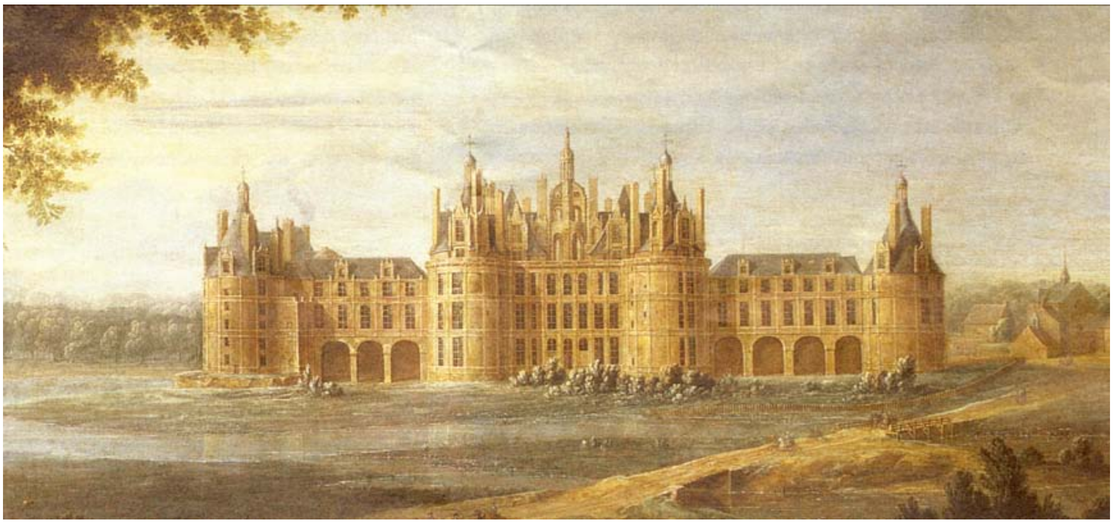


To add intrigue to the Pozzo di San Patrizio at Orvieto, we know that Sangallo the Younger had worked with, and learnt from, Leonardo da Vinci, who was his senior by 32 years We also know that Leonardo was a fan of Archimedes and Leonardo's notebooks include a drawing of a double helix staircase that looks remarkably like the Orvieto well.

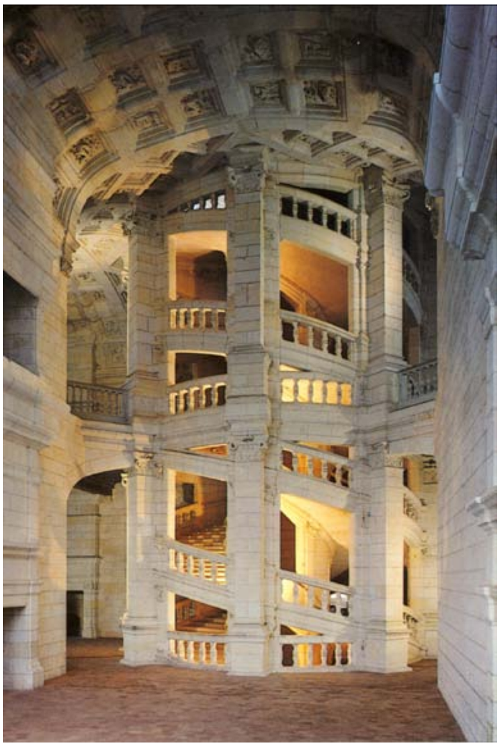
Drawing by Leonardo da Vinci
for a double helix staircase, or ramp.
(Paris Manuscript B, 1488-1490)

In 1915, Leonardo, aged 64, moved to the small castle of Cloux, near the royal residence of his new patron, Francis I, at Amboise on the Loire.

Francis I built the Chateau de Chambord, between 1519 and 1543, in part to be near his mistress, the Comtesse de Thoury. Chambord is centred on a wonderful double helix spiral staircase.



Chambord Chateau, Loire



Leonardo da Vinci's Double Helix Staircase in Chambord Chateau

It is almost certain that this is one of the few designs by Leonardo that reached fruition. There is some evidence that much of Chambord was based on a Leonardo design for a chateau at Romorantin, for the king's mother, but for me the double helix staircase is sufficient. This was Leonardo's last throw of the dice, he died on 2 May 1519 at the age of 67.

5. A FINAL TWIST

By the time I had finished the thought journeys that are documented in the previous chapters I had no doubt that the Sydney Opera House facility was the first time a double helix structure had been designed for such a purpose.

It was with this certainty that, in March 2007, on a visit to my son in Cairns, I removed the plastic wrapper from the Peter Jones biography of Ove Arup. Ove Arup was one of the most famous engineers of the 20th century and the crowning achievement of his consulting organisation (Ove Arup and Partners) was developing a structural method to implement Ulzon's sketch of the shells of the Sydney Opera House. My son, like all 5500 staff members of Arups, had been given a copy of the founders biography, but had not got round to reading it.

In the steamy heat of Cairns I browsed vaguely through the section describing Ove Arup's involvement in the Opera House and then couldn't believe my eyes when I saw Photograph No 32, reproduced below. What it showed was that in 1938, as part of designing air raid shelters in London, Ove Arup had designed an underground helical structure that was intended later to act as a car park. Well, I thought, at least he didn't think of the double helix. But I had only to turn to page 74 to find out that I was wrong. As Peter Jones says: "*In his March version, Ove designed ramps in the form of two helixes, one inside the other, echoing his design of the penguin pool at London Zoo.*" Unfortunately we have been unable to locate the drawings of the double helix version, but we accept Peter Jones' statement.

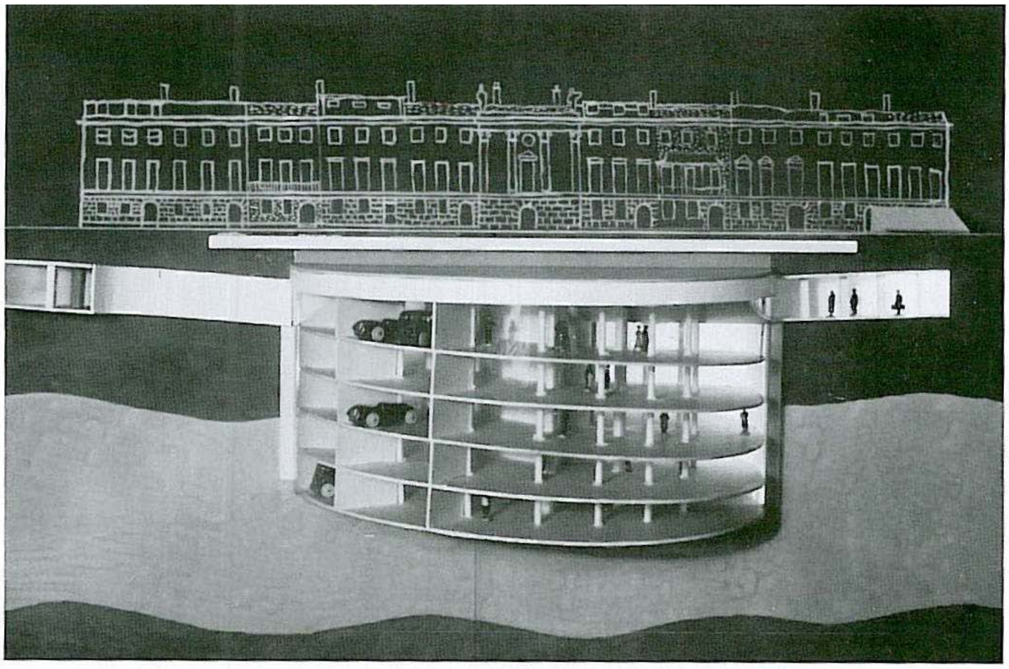


Figure 2: Model of Ove's design for helical underground shelter in Finsbury, intended for later use as car-parks, 1938.

So, incredible as it seems, the double helix structure of the car park of the Sydney Opera House, was created unknowingly by Messrs Colefax, Barry and Barrelle to almost exactly mimic a 55 year old, never-built, Ove Arup design. We can only wonder: "would Ove Arup personally have come up with the double helix design for the car park if he had been retained by one of the groups that submitted turnkey designs in response to the NSW Government tender of 1990?"

ADDENDUM

THE MATHEMATICS OF A HELIX AS GEOMETRY, AS A PUMP, A SPRING AND A PROPELLOR

1. GEOMETRY

The mathematics of a helix is very simple, yet the simplicity contains a wonderful elegance.

As illustrated in Figure 1, the x, y, z coordinates of a point on the surface of a helix, at a radius ‘r’ from the centre are defined as

$$x = r\cos\theta \qquad \text{... (1)}$$

$$y = r\sin\theta \qquad \text{... (2)}$$

$$z = \frac{p\theta}{2\pi} \qquad \text{... (3)}$$

where

θ = angle of rotation from the starting point, in radians
p = pitch of the helix, being the rise in one revolution
r = radius to the point

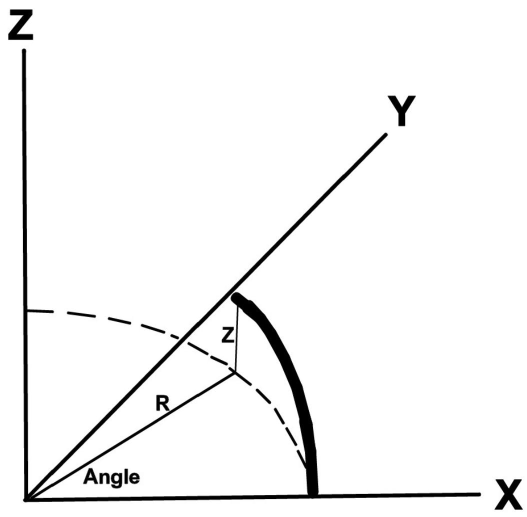


Figure 1: Helix Geometry

A horizontal cut through a helix is a horizontal line, and a tangent to a helix is at a constant angle to the axis.

The next question that may spring to mind is; what shape do I cut a sheet of paper, or steel or whatever, so as to make a single flight of a helix, i.e. one rotation? In one sense the answer is that this cannot be done. Because, although a helix is just an inclined plane wrapped around an axis, it cannot be created without distorting the material that is cut from the flat sheet. While a piece of a flat sheet can be tilted without doing internal work on the substance making up that sheet, internal work has to be done to deform a helix. However, it is easy, by hand, to input the necessary energy to form a piece of paper into a helix, and with a powerful machine this can be done to steel. The plan geometry of the piece to be deformed is determined as set out below.

To create a helix with the following properties;

Inside diameter	=	d
Outside diameter	=	D
Pitch diameter	=	P

it is necessary to first calculate the developed inside and outside diameters. These are:

- Developed outside diameter, **A =** $\sqrt{(\boldsymbol{\pi D^2}) + \boldsymbol{p^2}}$
- Developed inside diameter, **B =** $\sqrt{(\boldsymbol{\pi d^2}) + \boldsymbol{p^2}}$
- The width of the helix, **b =** $\frac{\boldsymbol{(D-d)}}{\boldsymbol{2}}$

To make this helix we must draw inside and outside radii of

r	=	(B x b)/(A – B)
R	=	r + b

Figure 2: First flight of mesh glued to pegs setting out the helix

Figure 3: Second flight of mesh glued into place

Figure 4: Completed single helix, two flights

To demonstrate this process to yourself you can make up a single helix using successive circular pieces as illustrated in the following photographs.

The elegance of the mathematics of a helix is taken to a quite extraordinary level in work of Sir Robert Stawell Ball (1840 to 1913) which he published in 1900 as *A Treatise on the Theory of Screws*. This amazing work deals with the mechanics (movements and forces) of solid bodies, eschewing any reference to our usual Cartesian (x, y, z) coordinate system. It is based on two well established theorems, which are actually facts. These are:

1. a free rigid body can be moved from any one position and orientation in space, to any other position and orientation, by a movement consisting of a rotation around a straight line, accompanied by translation parallel to that line (Chasles, 1830), and
2. any system of forces which act upon a rigid body can be replaced by a single force, and a couple in a plane perpendicular to that force (Poinsot, 1806).

Building on these statements, Robert Bell, builds his theory, first in simple English, but then followed by, to me, incomprehensible mathematics. His simple English is: the motion of any rigid body "is precisely the same as if it were attached to the nut of a uniform screw (in the ordinary sense of the word) which had an appropriate position in space, and an appropriate number of threads to the inch."

I have attempted to follow Ball's reasoning, and maths, many times, and each time advance a few pages further into the 500 pages of a 1998 reprint[1]. However, Ball clearly recognises that audiences would find his procedures obscure, and so in 1887 he gave a talk, to The Mathematical and Physical Section of the British Association, in the form of a delightful parable, involving a committee that included Mr Helix, Mr Cartesian, Mr Commonsense and others. It is a long parable, but the following extracts give an understanding of what Ball was on about.

> *There was once a rigid body which lay peacefully at rest. A committee of natural philosophers was appointed to make an experimental and rational inquiry into the dynamics of that body. The committee received special instructions. They were to find out why the body remained at rest, notwithstanding that certain forces were in action. They were to apply impulsive forces and observe how the body would begin to move. They were also to investigate the small oscillations. These being settled, they were then to ____ But here the chairman interposed; he considered that for the present, at least, there was sufficient work in prospect. He pointed out how the questions already proposed just completed a natural group. "Let it suffice for us," he said, "to experiment upon the dynamics of this body so long as it remains in or near to the position it now occupies. We may leave to some more ambitious committee the task of following the body in all conceivable gyrations through the universe."*
>
> *The committee was judiciously chosen. Mr Anharmonic undertook the geometry. He was found to be of the utmost value in the more delicate parts of the work, through his colleagues thought him rather prosy at times. He was much aided by his two friends, Mr One-to-One, who had charge of the*

[1] Cambridge University Press, Paperback 0-531-63650-7.

homographic department, and Mr Helix, whose labours will be seen to be of much importance. As a most respectable, if rather old-fashioned member, Mr Cartesian was added to the committee, but his antiquated tactics were quite out-manoeuvred by those of Helix and One-to-One. I need only mention two more names. Mr Commonsense was, of course, present as an ex-officio member, and valuable service was rendered even by Mr Querulous, who objected at first to serve on the committee at all. He said that the inquiry was all nonsense, because everybody knew as much as they wished to know about the dynamics of a rigid body. The subject was as old as the hills, and had all been settled long ago. He was persuaded, however, to look in occasionally.

The committee assembled in the presence of the rigid body to commence their memorable labours. There was the body at rest, a huge amorphous mass, with no regularity in its shape – no uniformity in its texture. But what chiefly alarmed the committee was the bewildering nature of the constraints by which the movements of the body were hampered. They had been accustomed to nice mechanical problems, in which a smooth body lay on a smooth table, or a wheel rotated on an axle, or a body rotated around a point. In all these cases the constraints were of a simple character, and the possible movements of the body were obvious. But the constraints in the present case were of puzzling complexity. There were cords and links, moving axes, surfaces with which the body lay in contact, and many other geometrical constraints. Experience of ordinary problems in mechanics would be of little avail. In fact, the chairman truly appreciated the situation when he said, that the constraints were of a perfectly general type.

In the dismay with which this announcement was received Mr Commonsense advanced to the body and tried whether it could move at all. Yes, it was obvious that in some ways the body could be moved. Then said Commonsense, 'Ought we not first to study carefully the nature of the freedom which the body possesses? Ought we not to make an inventory of every distinct movement of which the body is capable? Until this has been obtained I do not see how we can make any progress in the dynamical part of our business.'

Mr Querulous ridiculed this proposal. 'How could you,' he said, 'make any geometrical theory of the mobility of a body without knowing all about the constraints? And yet you are attempting to do so with perfectly general constraints of which you know nothing. It must be all waste of time, for though I have read many books on mechanics, I never saw anything like it.'

Here the gentle voice of Mr Anharmonic was head. 'Let us try, let us simply experiment on the mobility of the body, and let us faithfully record what we find.' In justification of this advice Mr Anharmonic made a remark which was new to most members of the committee: he asserted that, though the constraints may be of endless variety and complexity, there can be only a very limited variety in the types of possible mobility.

It was therefore resolved to make a series of experiments with the simple object of seeing how the body could be moved. Mr Cartesian, having a reputation for such work, was requested to undertake the inquiry and to report to the committee. Cartesian commenced operations in accordance with the well-known traditions of his craft. He erected a cumbrous apparatus which he called his three rectangular axes. He then attempted to push the body *parallel to one of these axes, but it would not stir. He tried to move the body parallel to each of the other axes, but was again unsuccessful. He then attached the body to one of the axes and tried to effect a rotation around that axis. Again he failed, for the constraints were of too elaborate a type to accommodate themselves to Mr Cartesian's crude notions.*

To him it appeared that the body could only move in a highly complex manner; he saw that it could accept a composite movement consisting of rotations about two or three of his axes and simultaneous translations also parallel to two or three axes. Cartesian was a very skilful calculator, and by a series of experiments even with his unsympathetic apparatus he obtained some knowledge of the subject, sufficient for purposes in which a vivid comprehension of the whole was not required. The inadequacy of Cartesian's geometry was painfully evident when he reported to the committee on the mobility of the rigid body. 'I find,' he said, 'that the body is unable to move parallel to z, or to y, or to z; neither can I make it rotate around x, or y, or z; but I could push it an inch parallel to x, provided that at the same time I pushed it a foot parallel to y and a yard backwards parallel to z, and that it was also turned a degree around x, half a degree the other way around y, and twenty-three minutes and nineteen seconds around z.'

'Is that all?' asks the chairman. 'Oh, no,' replied Mr Cartesian, 'there are other proportions in which the ingredients may be combined so as to produce a possible movement,' and he was proceeding to state them when Mr Commonsense interposed. 'Stop! stop!' said he, 'I can make nothing of all these figures. This jargon about x, y and z may suffice for your calculations, but if fails to convey to my mind any clear or concise notion of the movements which the body is free to make.'

Many of the committee sympathised with this view of Commonsense, and they came to the conclusion that there was nothing to be extracted from poor old Cartesian and his axes. They felt that there must be some better method, and their hopes of discovering it were raised when they saw Mr Helix volunteer his services and advance to the rigid body. Helix brought with him no cumbrous rectangular axes, but commenced to try the mobility of the body in the simplest manner. He found it lying at rest in a position we may call A. Perceiving that it was in some ways mobile, he gave it a slight displacement to a neighbouring position B. Contrast the procedure of Cartesian with the procedure of Helix. Cartesian tried to force the body to move along certain routes which he had arbitrarily chosen, but which the body had not chosen; in fact the body would not take any one of his routes separately, though it would take all of them together in a most embarrassing manner. But Helix had no preconceived scheme as to the nature of the movements to be expected. He simply found the body in a certain position A, and then he coaxed the body to move in any way the body liked to any new position B."

With the aid of a skilful mechanic he prepared a screw with a suitable pitch, and adjusted this screw in a definite position. The rigid body was then attached by rigid bonds to a nut on this screw, and it was found that the movement of the body from A to B could be effected by simply turning the nut on the screw. A perfectly definite fact about the mobility of the body had thus been ascertained. It was able to twist to and fro on a certain screw.

The success of Helix encouraged him to proceed with the experiments, and speedily he found a second screw about which the body could also twist. He *was about to continue when he was interrupted by Mr Anharmonic, who said, "Tarry a moment, for geometry declares that a body free to twist about two screws is free to twist about a myriad of screws. These form the generators of a graceful ruled surface known as the cylindroids. There may be infinite variety in the conceivable constraints, but there can be no corresponding variety in the character of this surface. Cylindroids differ in size, they have no difference in shape. Let us then make a cylindroid of the size, and so place it that two of its screws coincide with those you have discovered; then I promise you that the body can be twisted about every screw on the surface. In other words, if a body has two degrees of freedom the cylindroid is the natural and the perfectly general method for giving an exact specification of its mobility".*

A single step remained to complete the examination of the freedom of the body. Mr Helix continued his experiments and presently detected a third screw, about which the body can also twist in addition to those on the cylindroid. A flood of geometrical light then burst forth and illuminated the whole theory. It appeared that the body was free to twist about ranks upon ranks of screws all beautifully arranged by their pitches on a system of hyperboloids. After a brief conference with Anharmonic and One-to-One, Helix announced that sufficient experiments of this kinds has now been made.

With perfect lucidity Mr Helix expounded the matter to the committee. He exhibited to them an elegant fabric of screws, each with its appropriate pitch, and then he summarised his labours by saying, 'About every one of these screws you can displace the body by twisting, and, what is of no less importance, it will not admit of any movement which is not such a twist.' The committee expressed their satisfaction with this information. It was both clear and complete. Indeed, the chairman remarked with considerable force that a more thorough method of specifying the freedom of the body was inconceivable.

The discovery of the mobility of the body completed the first stage of the labours of the committee, and they were ready to commence the serious dynamical work. Force was now to be used, with the view of experimenting on the behaviour of the body under its influence. Elated by their previous success the committee declared that they would not rest satisfied until they had again obtained the most perfect solution of the most general problem.

And so the parable continues for many pages, and sets out how with three terms being defined for the operation of a helix, namely 'pitch', 'twist' and 'wrench', a complete mathematics for the mechanics of a rigid body of any shape can be defined without recourse to Cartesian coordinates.

In the early 20th century, Ball's work sank into obscurity, but was revived post-1960 when it was found to be of great practical importance to robotics.

2. HELICIAL SPRING

As noted in Section 1, above, it is not possible to cut a flat piece of material, and form it into a helix without imparting strain energy into the material, i.e. without distorting the material. Equally well, it is not possible to flatten a helix without expending energy. That is why steel helices make such good springs.

The common, round wire, helical compression spring is shown in Figure 5. The key parameters from the viewpoint of spring characteristics are the:

- Diameter of the wire : d
- Mean diameter : D
- Pitch ; P, or coil angle: α, and
- Number of active coils (360°), which may be less than the total number of coils if the end coils are closed and/or ground flat: N_t.

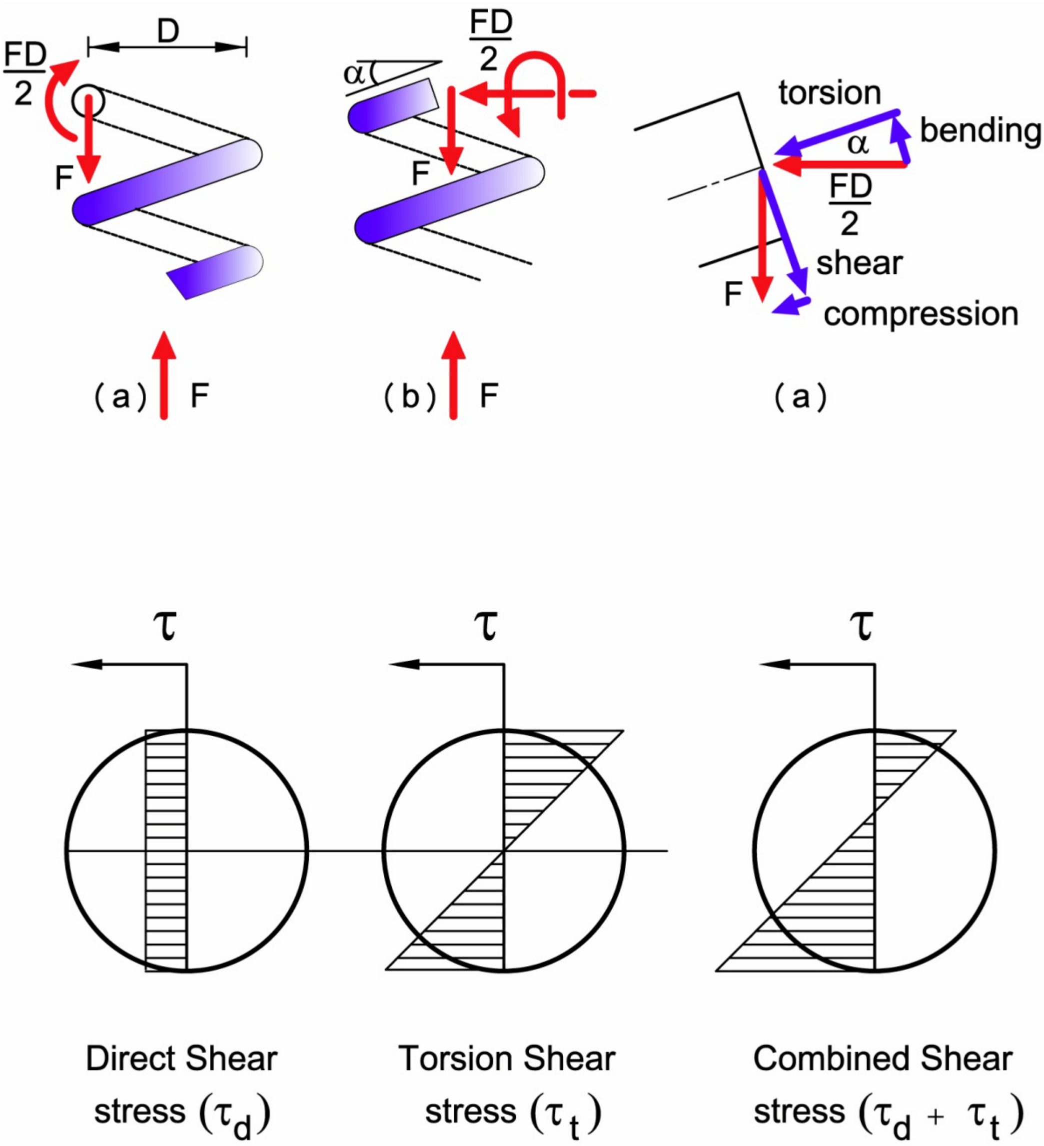


Figure 5: Forces and stresses in a helical spring

Figure 5 shows the load and stress distribution in a round wire helical spring under load. In order to maintain equilibrium there must be a shear force F in the wire and an equilibrating rotational moment FD/2. The enlarged section through the wire shows that there are four components of force on this surface, namely:

Shear Force	=	F cos α
Compressive Force	=	F sin α
Torque	=	$\frac{FD\cos\alpha}{2}$
Bending Moment	=	$\frac{FD\sin\alpha}{2}$

If the helix angle, α, is small, which is true for close coiled springs, then

sin $\alpha \approx 0$
cos $\alpha \approx 1$

Hence, only the shear force and torque are significant. The maximum combined shear stress at the inside surface of the wire is:

$$\tau_{max} = \frac{FD}{2} \times \frac{\frac{d}{2}}{\left(\frac{\pi d4}{32}\right)} + \frac{4F}{\pi d^2}$$

$$= \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

$$= \frac{8FD}{\pi d^3}\left(1 + \frac{d}{2D}\right) \qquad \text{... (4)}$$

This equation is written as

$$\tau_{max} = \frac{8FD}{\pi d^3} \times K$$

Where K is a stress factor and requires modification from the value of 1 + 0.5 d/D to allow for the fact that the curvature of the helical spring causes stress concentration and hence higher stresses than given by equation 4, above. This influence gives the following factor[2]

$$K = \frac{4c-1}{4c-4} + \frac{0.615}{C}$$

where

C = D/d (Spring Index)

The deflection of a spring under the force F is obtaining using Castigliano's theorem which states:

When forces act on elastic systems subject to small displacements corresponding to any force collinear with the force is equal to the partial derivative to the total strain energy with respect to that force.

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[2] Wahl, A M. Mechanical Springs, 2nd Ed, 1963. Reprinted by Spring Manufacturers Institute, 1993.

The strain energy includes that due to shear and that due to torsion.

Strain Energy U = $\frac{T^2 l}{2GJ} + \frac{F^2 l}{2AG}$

where

G = shear modulus ≈ 79GPa
T = FD/2
l = πDn
A = $\pi d^2/4$

Replacing T = FD/2, l = πDn, A = $\pi d^2/4$ the formula becomes

Strain Energy U = $\frac{4F^2D^3n}{d^4g} + \frac{2\,F^2Dn}{d^2G}$

Using Castiglianos theorem to find the total strain energy ...

$$\boldsymbol{\delta} = \frac{\partial U}{\partial F} = \frac{8FD^3n}{d^4G} + \frac{4FDn}{d^2G}$$

Substituting the spring index C

$$\boldsymbol{\delta} = \frac{8\,F\,C^3n}{Gd}\left(\mathbf{1} + \frac{1}{2C^2}\right)$$

In practice the term (1 + 0,5/C^2) which approximates to 1 can be ignored.

Hence the spring stiffness is

$$\boldsymbol{k} = \frac{F}{\delta} = \frac{Gd}{8c^3n}\left(\frac{C^2}{C^2 + 0{,}5}\right)$$

In practice the term (C^2/(C^2 = 0,5)) which approximates to 1 can be ignored, so

$$\boldsymbol{k} = \frac{Gd}{8c^3}\boldsymbol{n}$$

...(5)

Despite the simplifying assumptions, Equation 5 agrees well with experimental data.

3. HELICAL PUMP

Whether or not the helical pump was invented by Archimedes, or by an unknown Assyrian, it is universally known as the Archimedes Screw. We have already discussed Vitruvius's design for such a screw pump, a design that was presumably based on experimentation, and which was very difficult to build, because of its eight flights. Empirical design processes were used until recent times to build such screw pumps. It was found that the geometry of an Archimedes screw is governed by its outer radius, length and slope, and its inner radius, number of blades, and pitch of the blades. The external parameters are usually determined by the location of the screw and how much material is to be lifted. The internal parameters, however, are free to be chosen to optimise the performance of the screw.

In 2000, Chris Rorres published a detailed analysis of the functioning of an Archimedes screw[3], and developed the mathematics to allow optimisation of the design. However, before summarising his findings it is worth noting that it has long been recognised that there is a limit on how fast a screw pump can be turned. Nagel observed that the rotational velocity of a screw, in revolutions per minute, should not be more than $50/D^{2/3}$, where D is the diameter of the outer cylinder in meters. If the screw is rotated much faster, turbulence and sloshing prevent the cavities from being filled. The screw churns the water in the lower reservoir and does not efficiently lift it. This was obviously not a problem in Egyptian times, no matter how many slaves were employed.

Chris Rorres solved the tricky geometric calculation of the volume of water in one cycle of the screw, i.e. in one of the moving 'buckets' that transfers water up the rotating helix[4] (see Figure 6).

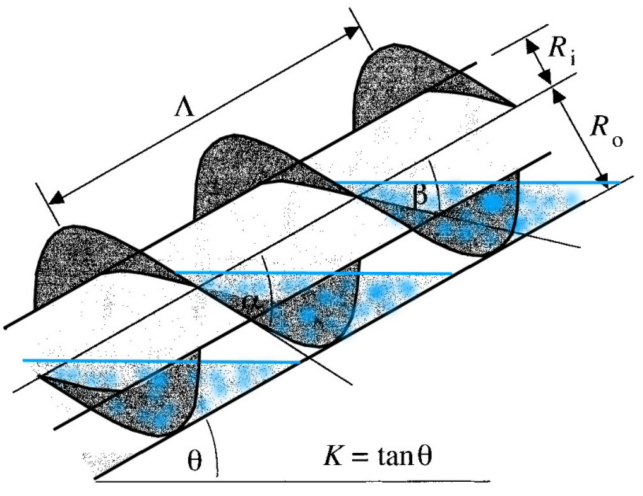

Figure 6: Figure from Rorres (2000)

The answer is complicated and results in the following equation:

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[3] Rorres, C. The Turn of the Screw: Optimal Design of an Archimedes Screw. Journal of Hydraulic Engineering, January 2000.

[4] A 'bucket' is "one of the maximally connected regions occupied by the trapped water within any one chute, where a chute is the region bounded by two adjacent blades".

$$\boldsymbol{v} = \frac{V_T}{\pi R_0^2 \Lambda} = \frac{N V_B}{\pi R_0^2 \Lambda} = \frac{N}{2\pi} \int_{\varphi_0}^{\varphi_1} \gamma_B(\varphi)\, d\varphi$$

where

V_T	=	volume of water in one cycle of the screw (m^3)
N	=	number of blades
R_0	=	radius of screw's outer cylinder
V_B	=	volume of one bucket
$\gamma_B(\varphi)$	=	a dimensionless parameter being a ratio of the cross-sectional area of the water in a bucket at a position along the screw, and the square of the outer radius.
Λ	=	pitch of one blade (m)

There is a restriction on the pitch of the screw, Λ, that is necessary for water to be trapped in the screw. This means that

$$\Lambda \leq \frac{2\pi Ro}{K}$$

where

K = the slope of the screw (m/m, i.e. dimensionless)

Having determined the equation for the volume of a 'bucket' and the limit on the pitch, for a given slope, Rorres summarises the problem as:

> *"Given the number of blades, outer radius and slope, find the inner radius and pitch that will maximise the volume of water emptied into the upper reservoir with each turn of the screw".*

The original paper can be referenced for details of the solution, which Rorres summarises in two figures, reproduced here as Figure 7, which give the optimal values of three items, namely:

1. the pitch ratio $= \frac{K\Lambda}{2\pi Ro}$
2. the radius ratio = $\frac{Ri}{Ro}$, and
3 the optimal volume per turn ratio = $\frac{V_T*}{\pi R_0^2 \Lambda}$, where

$$V_T* = \frac{1.52 R_0^3}{K}$$

and V_T* is the optimal volume per turn.

Rorres' calculations show that Vitruvius' design had an efficiency of about 83% of the optimal design. Not bad for an empirical Roman engineer.

Most modern screw pumps have 1, 2 or 3 blades, and for these cases the optimal design is summarised as per Figure 8.

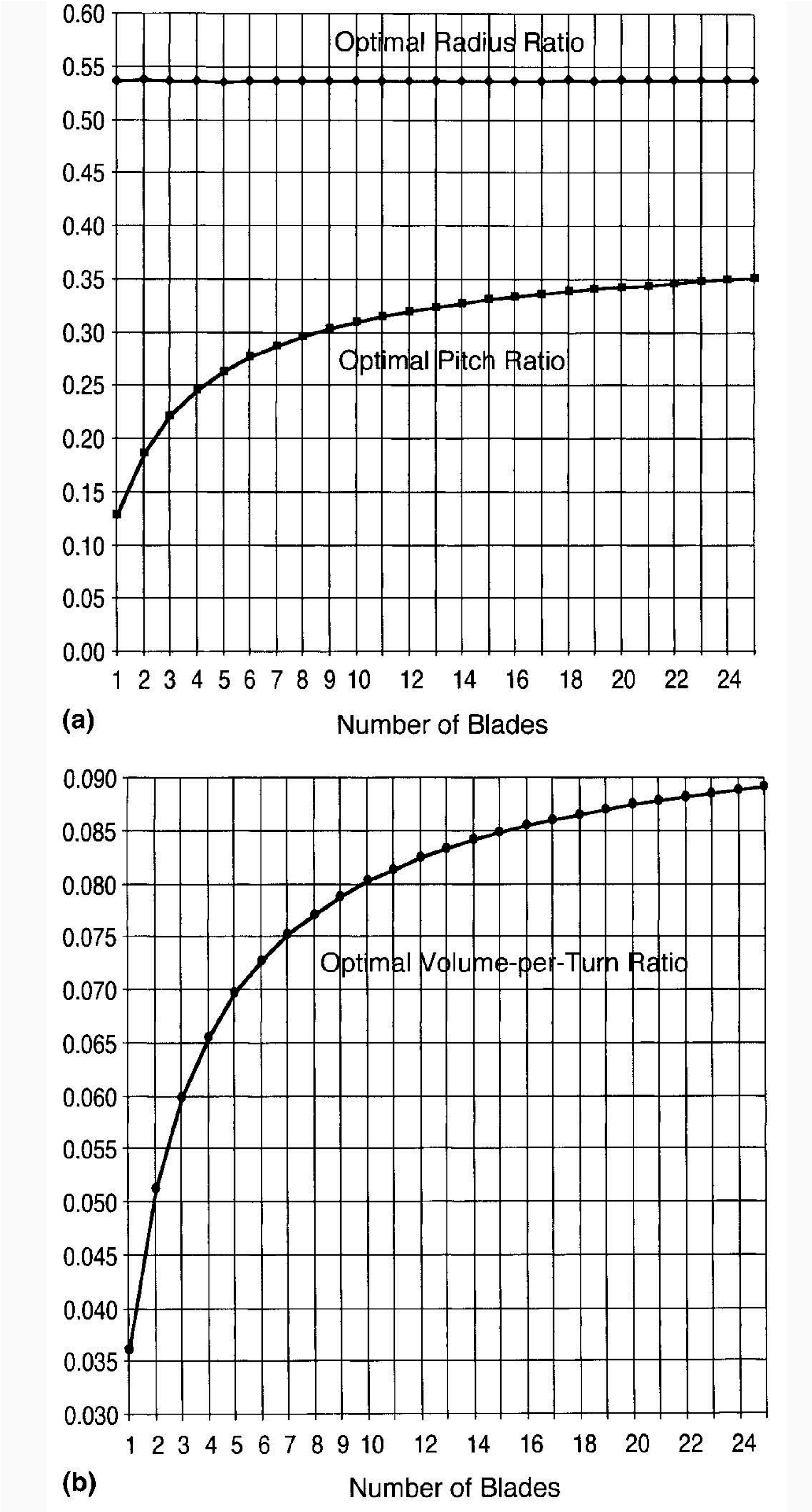

FIG. 10. Graphical Representation of Data in Table 1

Figure 7: Solutions by Rorres for Optimised Archimedes pump

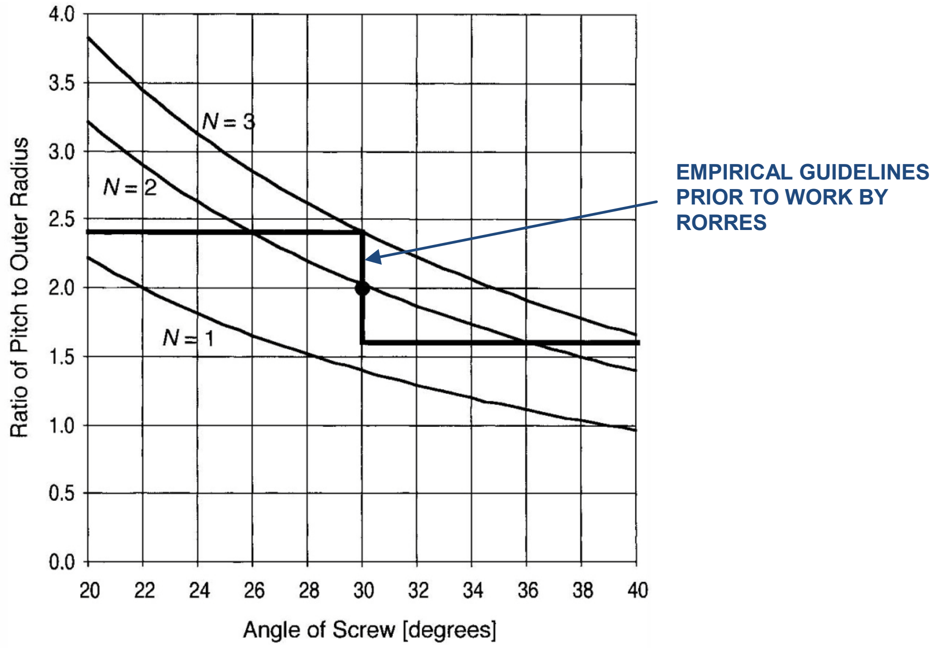

Figure 8: Optimised practical pump

4. THE SCREW AS A POWER GENERATOR

While the Archimedean screw has been used as a pump for thousands of years, it is only since the latter decade of the 20th century that it has been shown to be excellent for hydropower under low head difference conditions (Muller and Senior[5]).

A German company, Ritz-Atro is a leader in this field and their research (Hellman[6]) has shown the following advantage of this application of a helix.

- no control system – the screw matches itself automatically to the supply frequency and the water supply,
- the efficiency is greater than with comparable waterwheels and small turbines,
- flat stable efficiency gradient,
- robust, long wearing, trouble free,
- no cleaning, little maintenance,
- no fine screens necessary,
- little underground digging required in comparison to turbines,
- fish pass through the screw unhindered and unharmed,
- water is oxygenated.

[5] Muller, G and Senior, J (2009). Simplified Theory of Archimedean Screws. Journal of Hydraulic Research, Vol 47, No. 5.
[6] Hellman, H D (2003). Gutachten Wirkungsgrabestimmung Einer Wasserkraftschnecke. Fabrikat Ritz-Altro. Technical University Kaiserslautern.

Under full design flow a helical generator (hydrodynamic screw) has an efficiency of about 84%, but an important feature is that at only 30% of design flow the efficiency is still very close to 80%. The work by Muller and Senior has shown that the efficiency increases with decreasing screw angle and increases with the number of turns (flights).

At a head difference at 8m, and a flow of 1000 lit/sec, a hydrodynamic screw generates about 50kW. At half this flow, and a head difference of only 2m, a screw will generate about 5kW which is similar to a fairly serious scale wind generator. And so we find that a helix, that probably first found application in pumping water in Assyrian times, has found application as a small-is-beautiful, clean generator of electricity in our power hungry age.

Two Ritz-Atro GmbH Screw Power Generators in Germany

5. THE PROPELLOR

The earliest simple propeller was some form of windmill, like those made famous by the Dutch. Leonardo di Vinci designed a screw propeller for his helicopter, clearly based on the work on Archimedes that we know he read. He also designed an automatic roasting spit where the rotation of the meat was generated by a propeller turning by the rising hot air in the chimney (see Figure 9). What material he proposed for the propeller so that it would not catch fire is not known!

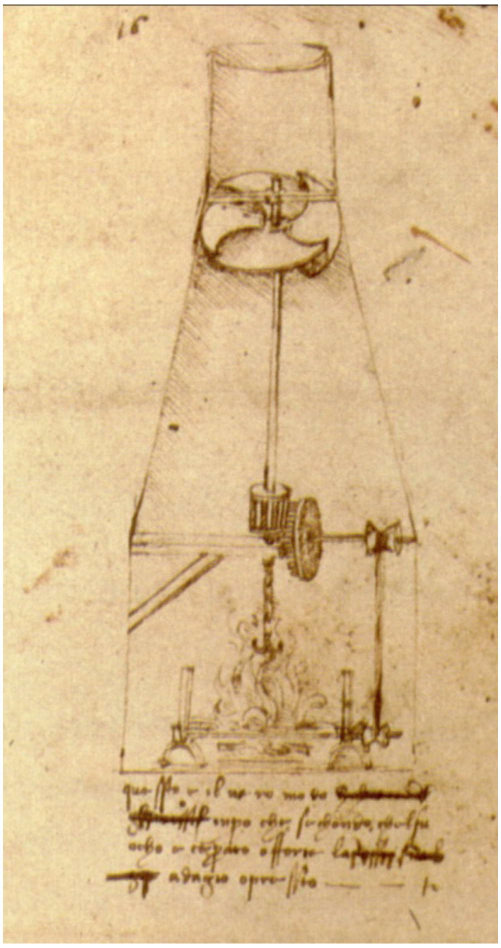

Figure 9: Leonardo's automatic spit roast; powered by the hot air turning a propeller

Leonardo was fascinated by the use of helical screws, work he built on earlier developments by the Sienese engineer, Francesco di Giorgio, Francesco produced numerous clever designs based on helices (see for example Figure 10), which are largely overlooked relative to those of Leonardo, mainly, I suspect, because Leonardo was such a brilliant draftsman, and also because he has had better press..

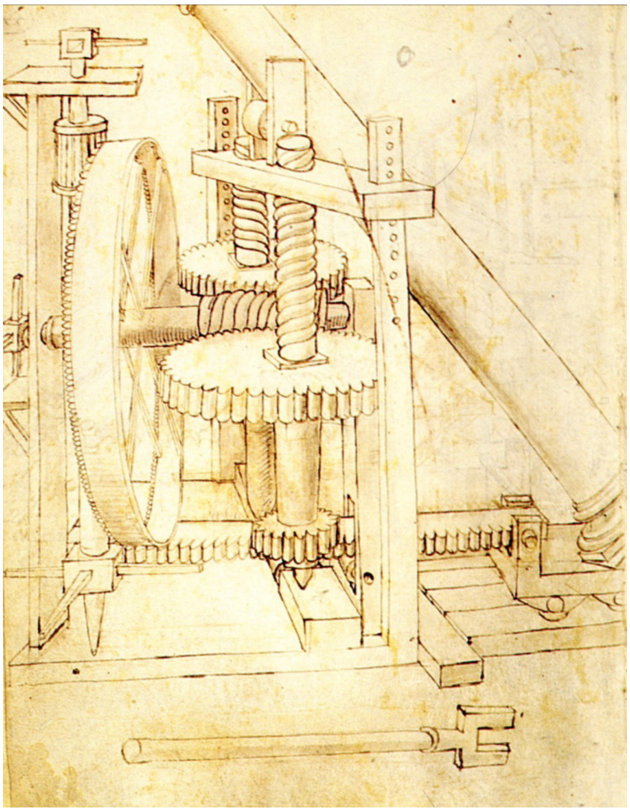

Figure 10: Column lifted by Francesco di Giorgio (1480)

There were some attempts in the late 1700's to use crude helical propellers to power vessels. For example, David Bushnell's 1775 submarine, *The Turtle*, was powered using hard operated single flight helices for vertical and horizontal movement.

It was the development of steam engines in the late 1700's, and particularly the application of such power in driving ships, that saw the major developments in propellers based on helices, or parts of helices. Although James Watt is credited as being the first to fit a screw propeller to a steam engine, most of the early steam ships were fitted with paddle wheels. These were inefficient, and between about 1825 and 1840 there were many patents for various types of propeller, all based on parts of helices (see Figure 11).

One of the more interesting contributions to this development process was by Sir Thomas Livingston Mitchell, then Surveyor General of New South Wales[7]. He acquired a British patent[8] in 1848 for the 'Bomerang[9] Propeller', which he had started experimenting with in 1836.

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[7] Sir Thomas Livingston Mitchell was also an explorer and collector of geological and botanical specimens (J.H.L Cumpston, "Thomas Mitchell", Webb, 1954).

[8] Sir Thomas acquired his patent through a London agent, Joseph Blunt, for this "improvement in propelling vessels" (Judy Wing, 1966. A History of the Patent Profession in Australia).

[9] This is how he spelled boomerang

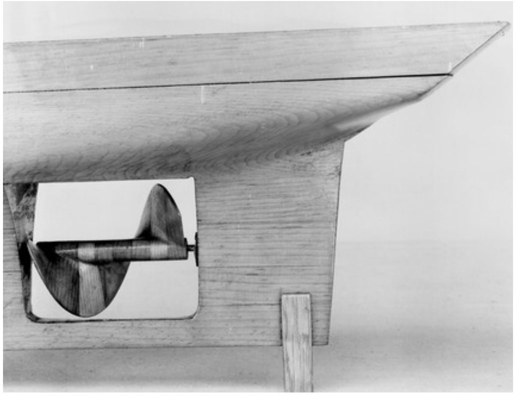

Figure 11: Screw propeller on vessel 'Archimedes' – patented in 1839 by Francis Petit Smith

Sir Thomas was a brilliant, irascible, workaholic, who was educated in the classics, showed precocious talent as an artist and geometer at the age of 9, could read Latin, Greek and French, and clashed with every governor of New South Wales between his arrival in 1827 and death in 1855. It is best to let Sir Thomas describe his work on what he called the Bomerang[10] Propeller using the draft notes he prepared for a presentation to the Australian Society on 30 December 1850.

Of all the novelties presented by New Holland, or New South Wales to the European, the original human inhabitant has always appeared to me by far the most interesting. Could he but tell us his history. What may be gathered from his language? Is there anything occult amongst his coradjes (or priests), handed down by tradition or can we learn anything from his arts? Seeing how simple and yet efficient his tools and appliances are, Nature alone, or his Maker, must have taught him these when the Australian man first began to exist. How ancient then may these weapons be; so few in number yet so efficient!

The spear and bomarang are available either in war or the chase- although the club seems chiefly for warlike purposes. The missiles are nicely adapted to the resistance of fluids, and the law of gravitation! Even in the form of clubs the centre of gravity seems to have been most fully considered.

But it is in the use of such missiles and clubs that these children of nature show how well they know her laws. By means of the woomera (wammerah), or throwing stick, the spear is thrown with much greater momentum, and, of course elevated velocity. The angular club, the rotary shield, the elastic handle of the stone hatchet all appear very original, but yet strictly consistent with whatever science teaches, and not susceptible of improvement by anything to be learnt at colleges.

The bomarang is one of the most remarkable of these missiles. Its flight through the air, from the hand of an Australian native seems in strict obedience to his will. In its return after a very varied course to the foot of the thrower, this weapon seems so extraordinary that a Vice President of the Royal Society (Bailey), about twelve years ago, observed to me "that its path through the air was enough to puzzle a mathematician".

[10] His spelling of boomerang

Such a remark by one of the ablest mathematicians of his time was not forgotten. On the contrary, it was remembered on the next occasion when I had opportunity of studying the flight of bomerangs thrown by the hands of Australian Aborigines, and then I perceived that in its rotary motion through the air, a hollow centre of greater or less diameter, but usually of about one third of the disc, was described by the whirl of the boomerang and it occurred to me that the centre of the whirling motion might be found in a line of equilibrium which should divide the surface acting on the air into three portions in such manner as that the eccentric portions should equal the central one. The discovery of this centre, insignificant as it may appear, was still something new, for on attaching a centre to a boomerang, it was possible to show that this centre was not only driving its rotary motion, the centre of that motion, but also the centre of gravity when in the state of rest, while it was apart from and quite clear of every part of it.

The natives when bent on exhibiting the more curious flights, twist the boomerang, by placing it at the fire, evidently for the purpose of giving it the property of spiral movement, thus showing how well they understand the screw-action upon the air. On making a small wooden model with a spiral turn like a screw, and giving it by means of an attached centre, and the fork and cord of a humming-top, rapid rotary motion, the model ascended to the roof of the room with such force as to be broken in pieces against it. Thus far I had proceeded in my study of the bomerang, when I last went to England on leave of absence.

There is much in discovery to reward and encourage the most patient investigations; but that any new application of a mechanism power could be devised from such a source, I always doubted myself. I considered the property I had discovered in the bomerang worth keeping secret. It was known in those days to one of my sons only, a native of the colony, and it died with him.

On Tuesday 11th January 1848, after having dined with the Master and Wardens of the Stationers' Company of the City of London[11]; I met at a Ball in Upper Harley Street, Mr Brunel, and after asking various questions about sails and sailways, I enquired whether the principle of the screw had yet been fairly applied to water? He replied "Not at all? There is still a great discovery to be made there?"

Like Bailey's remark made ten years before, which led first to my study of the boomerang, this reply of Brunel determined me to try experiments; and I bespoke a set of driving wheels to be applied to a bomerang propeller in a boat.

A lecture I heard at the Royal Institution in March of the same year, by Professor Cowper – on paddle wheels and propellers – was concluded in these words – "Until the propeller can be made so as to be free from lateral resistance, great speed cannot be attained by this means, but whoever invents a propeller which shall not have this defect, will make a very great discovery".

That remark determined me to take out a Patent, having been adapted to sail for Australia before I could complete my experiments. The result of these experiments (for which have since had leisure) and of much subsequent study, I now lay before the Australian Society.

In order to continue my history of this investigation, it is in keeping that I should state that having had a boat built at Sydney and attached a small boomerang propeller to

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[11] The worshipful Company of Stationers and Newspaper Makers, London. Established 1403.

the boat, and worked it with the driving wheels, turned by two men, the results induced me to send to England for a small steam engine which, with the obliging attention of Mr Daniel Cooper Senior, of 3 Corphall Chambers, arrived in due course.

The power of this engine being only that of three men, and equal only to two men, when the power of the third, lost by friction, was considered; was not great enough for such display of speed as I could have wished – although it was sufficient and perhaps even the best, for testing the accurate working of the bomerang on the water. The boat was ill-adapted for the purpose, very crank[12] and so leaky that the wheels worked partly in the water – nevertheless the result was surprisingly satisfactory.

Sir Thomas did not have the understanding of hydrodynamics to analyse and design a propeller, but then nobody at that time had integrated the pressure differential work of Bemoulli, with the force equilibrium of Newtown, to understand airfoil behaviour. Now we understand that a propeller in air is just a special spinning wing, although a propeller in water is somewhat different. However, by his experiments with Boomerangs Sir Thomas determined that the Australian Aboriginals had discovered airfoil lift many thousands of years before Bemoulli. It is also clear from his other writings that Sir Thomas recognised the limitations of the screw propeller that had been patented by Sir Francis Petit Smith in 1836 (see Figure 11) and demonstrated on the 237 ton steam yacht Archimedes. In a handwritten note Sir Thomas wrote as follows:

> *"In the Archimedes, where the screw was first employed for the propulsion of vessels – the single term screw was substituted for the original screw of three turns; the decision of the single turn into two, four, six and eight parts or segments was next tried, and the two-threaded[13] screw, consisting of two half turns.*
>
> *Still there was lateral resistance; and what was called "choking of the centre" – hence the use of rotary paddles composed of sections of screws, and the form last adopted similar to the two vanes of a windmill, as used in the Great Britain."*

In 1853, Thomas Mitchell published a substantially extended version of the notes for his Australian talk that are quoted above. The booklet is titled "Origin, History and Description of the Bomerang Propeller", being a lecture delivered to the United Services Institution, London on 22 June 1853.

The illustrations in that booklet provide a clear understanding of the design. His Figures 1 and 2, reproduced herein as Figure 12, show that his design comprised a boomerang draped over a helix. He then shows a sketch of the first boomerang "ever applied to water" (see Figure 13). It must have generated substantial vibrations because of its eccentricity around the shaft. However, he partly solved this problem by solving another problem; to quote:

> *"But it was soon ascertained that none of the apertures in the dead wood of vessels would admit the Bomerang, as one blade, and it would be necessary to divide it into two blades. This could easily be done, still preserving all its properties of equilibrium, obliquity, concavity and convexity, by attaching the*

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[12] **Crank** *adj.* Liable to lurch or capsize, as a ship. *The Macquarie Dictionary*
[13] A double helix.

blades to the shaft in the same relative position, only attaching then so that they occupied but half of the fore-and-oft space. This will easily be understood by any of these models." (see Figure 14).

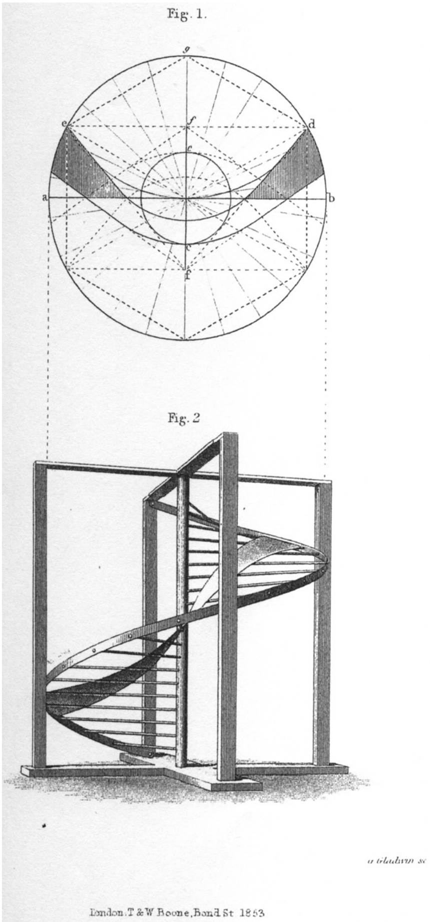

Figure 12: The Design Concept for the "Bomerang Propeller"

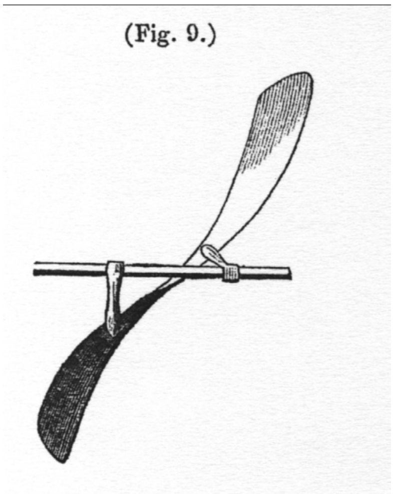

Figure 13: The First "Bomerang Propeller"

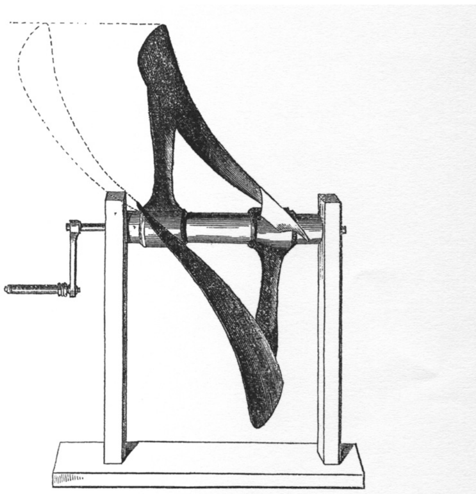

Figure 14: Method of Shortening the "Bomerang Propeller"

He then proceeds to describe a very impressive test of the propeller on a ship. *The Keera* in Sydney Harbour (see Figure 15). In his words:

"In the remote colony where by official duties obliged me to remain, four years had elapsed after I had taken out a patent, and no prospect appeared even then, of my having ever an opportunity of attaching to any vessel of sufficient dimensions to test on a large scale, a propeller which had afforded such encouraging results with a boat.

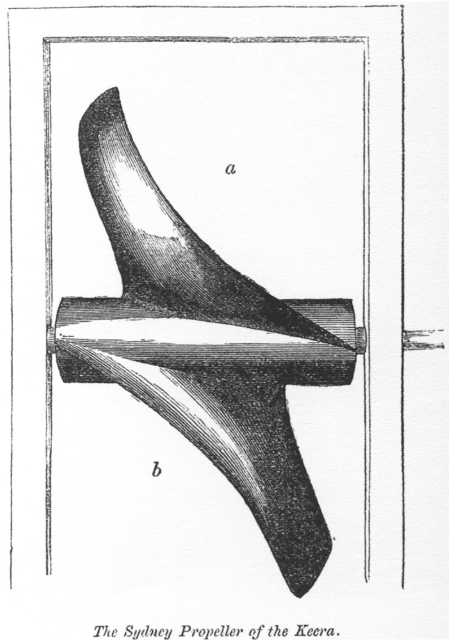

Figure 15: The propeller of the Keera

At length the "Keera" arrived in Sydney from England, with engines of 70-horse power, and a three-bladed screw, driven by a multiple of 3. The diameter of the propeller was 5 ft. 8 inches; the pitch 8 feet. With this propeller the vessel could be made to attain only an average speed of 6 or 7 knots per hour. The owners, Messrs. Smith & Co., allowed me to apply a Bomerang Propeller (cast at Sydney by Mr Struth,) of the same diameter and pitch as the "Keera's" own screw, which had 267 square inches more surface than the Bomerang Propeller made for this trial. The result was remarkable, and the local government and the public of Sydney, felt so much interest in it, that I was allowed leave of absence from my office, to come to England, that I might introduce this invention in the country to which they have to look across seas so vast that any method of crossing them more rapidly, appears to them important.

The Keera, with the Bomerang Propeller at her stern, passed over one knot or geographical mile, in port Jackson, in five minutes, which was exactly twelve knots an hour; and, in a trip down the harbour four or five miles, her speed corresponded to this rate – thus nearly doubling the speed obtained by her English propeller, which never exceeded seven knots an hour."

In 18 Mitchell was granted leave of absence so as to market his invention in England and so finally make his fortune. His tests, trials and tribulations in England are too complex and extensive to do justice in this document, but suffice it to say he had some very large propellers made in Liverpool, constrained in length by the available spaces in the test ships, met mixed success, and had to return to New South Wales before cracking the market and making his fortune.

I do not know whether anybody in modern times has properly tested, or analysed Sir Thomas Mitchell's Bomerang Propeller but it does seem to me that the modern propellers on 'silent' submarines bear an uncanny resemblance to Sir Thomas' designs.

Turning to the 21st Century, the designs of propellers for aircraft, ships, silent submarines and wind generators, have become very sophisticated and are based on extensive computer and physical modelling. The mathematics of lift, thrust and torque is beyond the scope of this document, but let it be noted that silting behind all the designs is the basic form of the helix.