Basic Soil Parameters.

Grading and Density

n = Porosity = $\frac{\text{Volume of Voids}}{\text{Total Volume}}$ = $\frac{V_v}{V_v + V_s}$ = $\frac{e}{1+e}$

e = Void Ratio = $\frac{\text{Volume of Voids}}{\text{Volume of Solids}}$ = $\frac{V_v}{V_s}$ = $\frac{n}{1-n}$

γ = Density of Soil = $\frac{\text{Weight of Soil}}{\text{Volume of Soil}}$ = $\frac{V_s G_s \gamma_w + V_w \gamma_w}{V_v + V_s}$

S_r = Degree of Saturation = $\frac{\text{Volume of Water}}{\text{Volume of Voids}}$ = $\frac{V_w}{V_v}$

ω = Water Content = $\frac{\text{Weight of Water}}{\text{Weight of solids}}$ = $\frac{\gamma_w V_w}{G_s V_s \gamma_w}$

From these definitions a number of useful relationships can be derived

A: Water content, degree of saturation & void ratio

$$\omega = \frac{e S_r}{G_s}$$

B: Bulk Density

$$\gamma = \frac{G_s + e S_r}{1+e} \gamma_w$$

Hence dry density

$$\gamma_d = \frac{G_s}{1+e} \gamma_w$$

C: Submerged Density

$$\gamma' = \frac{G_s - 1}{1+e} \gamma_w$$

D: Dry density, bulk density & water content

$$\gamma_d = \frac{\gamma}{1+\omega}$$

E: Degree of saturation, bulk density & water content.

$$S_r = \frac{\omega}{\frac{\gamma_w}{\gamma}(1+\omega) - \frac{1}{G}}$$

Grading of Soils

The clay fraction is generally accepted as being smaller than 2μ ie $\cdot 002$ mm.

Silt : 0·002 - 0·06 mm

Sand : 0·06 - 2 mm

The Uniformity Coefficient $U = \frac{D_{60}}{D_{10}}$

Atterberg limits

With regard to remoulded clays the numerical position of the water content with respect to the liquid and plastic limits is evidently an approximate measure of strength

$$\text{Liquidity Index} = \frac{\omega - PL}{LL - PL} = L.I.$$

Cassagrande's classification makes use of these limits

Inorganic Silts : M — Plot below A line.

Inorganic Clays : C — Plots above A line

Organic Clays & Silts : O — Plots below A line

If the LL is $> 50\%$ then it is high "H"

" " " " $< 50\%$ " " " low "L"

The "Activity" of Clays

The ratio $\frac{PI}{\text{Clay Fraction}}$ is defined as the activity of the clay, it being related to the mineralogy and the geological condition of deposition of the clay.

Permeability

With the exception of flow through coarse gravels the movement of water through soils is streamlined and can be represented by Darcy's law

$$v = k \frac{\Delta h}{\Delta l}$$

where $k = \frac{K}{\eta} \gamma_w$ where K = Permeability : cm^2
k = Coefficient of permeability cm/sec

The value of k thus depends on the temperature of the water

Thus $k_1 \eta_1 = k_2 \eta_2$ and the value is usually quoted for 10°C. or 20°C.

The Principle of Effective Stress.

A soil may be visualized as a compressible skeleton of solid particles enclosing voids, which in saturated soils are filled with water, and in partly saturated soil with both air and water. Shear stresses are carried only by the skeleton of solid particles, except at very high rates of strain.

The total normal stress on any plane is in general the sum of two components 1/ Stress carried by the soil particles

2/ The pressure of the fluid in the void space.

The only part which controls volume change and the frictional component of shear strength is the interparticle stresses.

For soils containing a single fluid

$$\sigma' = \sigma - u$$

For soils containing two fluids - air & water

$$\sigma' = \sigma - u_a + \chi (u_a - u_w)$$

Volume Changes.

For equal all round changes in stress

$$\Delta V / V = - C_c \, \Delta \sigma'$$

Shear Strength

Mohr Coulomb failure criteria for effective stress

$$\tau_f = c' + (\sigma - u)\tan\phi'$$

c' = Cohesion intercept } with respect to effective stress
ϕ' = Angle of shearing resistance } with respect to effective stress

The expression of soil properties in terms of effective stress has two advantages

① The only satisfactory basis for understanding the strength & deformation characteristics of soils

② Practical design methods are obtained and can be checked in the field by direct observation of pore pressure

Evidence to support the idea.

a: With respect to volume change

i/ Continued settlement of a foundation under constant load due to gradual dissipation of excess pore pressure

ii/ Regional settlements due to ground water lowering in compressible soils

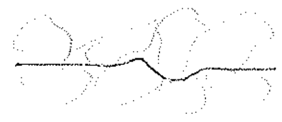
iii/ Settlement due to seasonal drying of compressible soils.

b: With respect to shear strength.

Easily shown by lab tests ie Cu test results
- no change in undrained strength since no change in effective stress under different confining pressures

Influence of area of contact between the particles

To consider the physical basis of the principle of effective stress



Consider forces acting across a surface in the soil which approximates to a plane but passes always through the pore spaces and points of contact

Let σ = total normal stress

σ_i' = average intergranular force per unit area of the plane

ie P_i = force per contact

$$\therefore \quad \sigma_i = \frac{\Sigma P_i}{\text{unit area}}$$

u = pore pressure

a : effective contact area per unit area of the plane

For equilibrium across the plane

$$\sigma = \sigma_i' + (1-a)u.$$

$$\therefore \sigma_i' = \sigma - (1-a)u$$

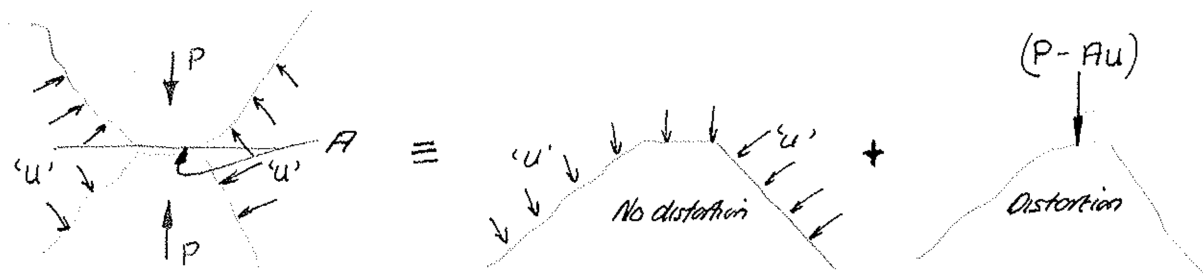
$$\text{ie} \quad \sigma_i' = (\sigma - u) + au$$

If σ_p = contact pressure between particles $\sigma_i = a\sigma_p$ and 'a' cannot be zero.

In practice "a" is small ie $\approx 0.5\%$ but can become meaningful under very high pressures or with large contacts

Effect w.r.t volume change.

Consider an enlarged view of the contact between two particles



P = average force per contact

N = ave. number of contacts per unit area

A = area of a particular contact.

The intergranular force per unit area of cross-section = σ_i'

$$\sigma_i' = NP$$

Now due to an all round pressure of 'u' no distortion of the particle can take place, if the material is isotropic. There is a very small decrease in volume but this can be ignored.

It is that part of the local contact pressure in excess of "u" which causes deformation of the soil structure ie the component $P/A - u$

Define σ_c' = component of normal stress which causes volume change due to deformation of the soil structure

$$\therefore \quad \sigma_c' = N \cdot A \left\{ P/A - u \right\}$$

$$= NP - uNA$$

If $a = NA$ then $\sigma_c' = NP - ua$

$$= \sigma_i' - au$$

But $\sigma_i' = (\sigma - u) + au$

$$\therefore \quad \underline{\sigma_c' = (\sigma - u) + au - au = \sigma - u.}$$

Thus although the intergranular force per unit area (intergranular stress) depends on the contact area 'a' in the presence of pore pressure, the volume change in the soil due to deformation of the particles depends simply on the difference $\sigma - u$ = Effective Stress.

<u>TEST VERIFICATION</u> Laughton 1955.

Requires the use of very high pore pressures to increase the value of the au term and large effective stresses to increase the contact area.

A sealed oedometer capable of very high pressures was used as follows.

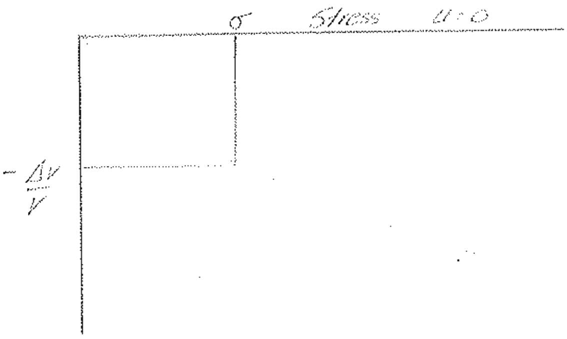
1/ Application of a known increment of total stress ($\Delta\sigma$) and pore pressure (Δu), for example such that $\Delta u = \frac{1}{4}\Delta\sigma$. Time was then allowed for consolidation, and the volume change noted.

2/ With σ constant, u is first made equal to zero and when the additional consolidation is complete the volume change is again noted.

$\sigma' = \sigma - u$ (1) OR $\sigma' = (\sigma - u) + au$ (2)

$$\frac{\Delta v}{v} = -C_c(\Delta\sigma - \Delta u) \quad \text{OR} \quad \frac{\Delta V}{V} = -C_c\left\{(\Delta\sigma - \Delta u) + \Delta u \cdot a\right\}$$

For a series of such increments we can relate volume change to stress for zero pore pressure



From this curve we can read off the stress σ' which gives the same volume change as any particular combination of σ & u

Eq^n 1 : $\frac{\sigma - \sigma'}{u} = 1.0$

Eq^n 2 : $\frac{\sigma - \sigma'}{u} = 1 - a$

The volume change under a certain σ & u is known ①
The volume change equivalent under a certain σ' is obtained from the curve.

Thus by calculating $\frac{\sigma - \sigma'}{u}$ the results showed that even for $a = 95\%$ of total area the correct equation was

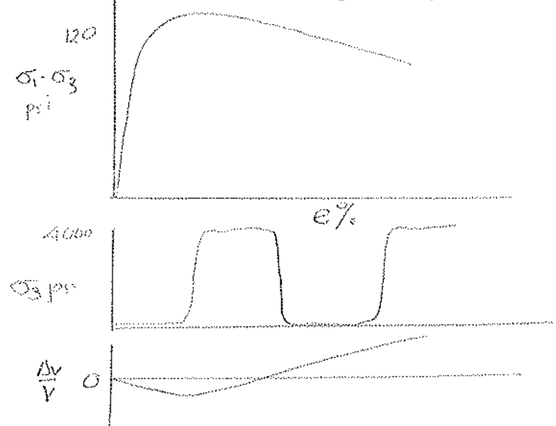
$$\sigma = \sigma' + u.$$

Effect w.r.t shear strength. Bishop, Webb & Skinner: 1965

High pressure triaxial cell as follows.
A balanced ram was used.

The confining pressure σ_3 and the pore pressure u were changed simultaneously by nearly 4000 lb/sq in

three times during the progress of a compression test while maintaining the difference $(\sigma_3 - u)$ constant at only 52.6 lb/sq in. No significant influence on the strength can be detected between the various sections of the curve for which the pressures were steady. Equally the volume change, which was corrected for the compressibility of the soil grains show no discontinuity



Effective stress in partially saturated soils

Air and water exists in the pore space at different pressures due to surface tension.

$$\sigma' = \sigma - u_a + \chi (u_a - u_w)$$

χ is a parameter which is $\simeq 1.0$ in fully saturated soils and $= 0$ in dry soils.

For intermediate soils χ will depend primarily on the degree of saturation S_r but will be influenced by the cycle of wetting and drying leading up to that value of S_r.

It must be noted however that clays are virtually different materials when air is present. The air in the voids actually changes the physical characteristics. Thus it is not so easy to consider χ.

For clays $u_a - u_w$ may be several atmospheres at lower degrees of saturation. It is difficult to measure as it tends to suck the water out of the ceramic used to measure it.

Measurement of pore pressure.

A ceramic disc of high air entry value can be used

Aerox Celloton grade VI : $k = 2.9 \times 10^{-6}$
air entry = 30 psi

Doulton Grade P6A $k = 2.1 \times 10^{-7}$
air entry = 22 psi

With a disc of consolidated and fired kaolin an air entry value = 45 psi was obtain. This value increased with time to 60 psi

For measuring pore air pressure a single layer of glass fibre cloth can be used. This has a lower attraction for water than the soil sample. Its volume together with that of its air filled connection to the null indicator should be small, since air is a compressible fluid and a compensating adjustment to the null position would otherwise have to be made.

The possibility of measuring directly large negative pore pressure is limited by the inability of the water in the measuring system to carry tension for more than a brief period. This places a lower limit to direct measurement of -1 atmosphere, or over long periods about -13 psi at sea level. However indirect measurements can be extended beyond this range by artificially increasing the 'atmospheric pressure' under which the test is being run. An air pressure is applied to the end of the sample. If the air pressure is equal to the cell pressure $\sigma_3 = u_a$ then one has the lower limit of effective stress for the given water content. Higher ~~air~~ pressures in the cell than in the pores will then give higher effective stresses.

<u>Pore pressure changes under undrained conditions</u>

Saturated Soil.

When an all round pressure is applied to a saturated soil from which drainage is prevented, the proportion of the stress carried by the pore water and by the soil structure depends on their relative compressibilities

The compressibility of soil structure $\quad C_c = \frac{-\partial V}{V} \times \frac{1}{\delta\sigma'}$

The compressibility of pore water $\quad C_\omega = -\frac{\partial V}{V} \times \frac{1}{\delta u}$.

The initial volume of an element of soil $= V$ of porosity $= n$.

$\therefore$ Volume of soil structure $= V$

Volume of water $= nV$

The decrease in volume of structure due to change in effective stress $= C_c \times V \times \delta\sigma'$

& pore fluid change $= C_\omega \times nV \times \delta u$

For zero drainage these must be equal

$$\therefore \quad C_c \times V \times \delta\sigma' = C_\omega \times nV \times \delta u.$$

$$\therefore \quad \delta\sigma' = \frac{C_\omega}{C_c} . n\delta u$$

But $\quad \delta\sigma' = \delta\sigma - \delta u$.

$$\therefore \quad \delta\sigma - \delta u = \frac{C_\omega}{C_c} . n\delta u.$$

$$\therefore \quad \delta u = \frac{1}{1 + n\frac{C_w}{C_c}} \delta\sigma$$

which is written $\delta u = B\,\delta\sigma$

and since $n\frac{C_w}{C_c} \to 0$ for most soils we have $B \doteqdot 1.0$

If the compressibility of the grains is included then

$$\delta u = \left\{ \frac{1}{1 + n\frac{C_w - C_s}{C_c - C_s}} \right\} \delta\sigma$$

Partially saturated Soils

In all partially saturated soils the value of the compressibility of the pore fluid is much greater than the water in the sat. case - and is comparable to C_c

$\therefore$ B lies between 0 & 1.0.

An approximate solution may be obtained by calculating the equivalent compressibility from Boyles law for gases and Henry's law of solubility. As the relationships are non-linear practical problems are solved graphically.

V = Volume of element of partly saturated soil.

V_v = Volume of voids.

S = Degree of saturation (V_w/V_v)

p_o = Initial pressure of air in the voids when the sample is unconfined. This is usually taken as atmospheric but the actual air pressure may be lower (10 psi) in samples dry of optimum that are sealed immediatly after compaction.

H = Henry's coefficient of solubility. This is $\doteqdot 0.02$ volumes per unit volume of water at room temperature and is actually independant of pressure.

n_o = initial porosity of the soil.

Henrys law: At a given temperature the weight of gas which will dissolve in a given volume of liquid is directly proportional to pressure.

By Boyles law we have that PV = constant. Thus if the pressure is doubled the dissolved weight will be doubled but the volume would be halved. Thus the volume occupied by this weight of dissolved gas is the same at all pressures.

Volume of air in unconfined sample:

$$\left.\begin{array}{ll}\text{free air} : & (1-S)V_v \\ \text{dissolved air} : & SV_vH\end{array}\right\} \text{ at } p = p_0 \text{ abs.}$$

If the pressure is changed to $p = p(\text{abs})$ the total vol ~~free~~ air $= (1-S)V_v \times p_0/p_{abs} + SV_vH \times p_0/p_{abs}$

$$\text{dissolved air} = SV_vH$$

Thus volume of free air $= \left\{(1-S)V_v + SV_vH\right\} p_0/p - SV_vH$.

If ΔV is the corresponding change in volume of the element and we neglect the compressibility of the soil particles then

$$\Delta V = \text{difference between two volumes of free air}$$

$$= \left[\left\{(1-S)V_v + HV_vS\right\} p_0/p - SV_vH\right] - (1-S)V_v$$

$$= \left\{(1-S)\,p_0/p + HS\,p_0/p - SH - (1-S)\right\} V_v$$

$$= \left\{p_0/p - 1\right\}\left\{1 - S + SH\right\} V_v.$$

$$\therefore \quad \frac{\Delta V}{V} = \left(p_0/p - 1\right)\left(1 - S + SH\right) V_v/V$$

$$= n_0 \left\{p_0/p - 1\right\}\left\{1 - S + SH\right\}$$

In order to write this in terms of an equivalent compressibility we must have in the form.

$$\Delta V/V \times \frac{1}{\delta p} = \frac{\Delta V}{V} \times \frac{1}{p - p_0} = f(n, S, H)$$

$$\text{ie } \Delta p = p - p_0.$$

Now $$\frac{p_0 - p}{p} = \frac{\Delta V}{V} \frac{1}{n_0(1-S+SH)}$$

BUT $$\frac{\Delta p}{p_0} = \frac{p - p_0}{p_0} = \frac{-\frac{p_0 - p}{p}}{\frac{p_0 - p}{p} + 1}$$

Thus $$\Delta p/p_0 = \frac{-\Delta V/V \frac{1}{n_0(1-S+SH)}}{\frac{\Delta V}{V} \cancel{n_0} \frac{1}{n_0(1-S+SH)} + 1}$$

$$\boxed{\frac{\Delta p}{p_0} = \frac{-\Delta V/V}{\Delta V/V + n_0(1-S+SH)}} \quad \text{— a.}$$

Thus we have the change in pore air pressure in terms of the change in volume and the initial conditions.

This equation is valid until the sample becomes saturated ie all the air is in solution

ie until $-\Delta V =$ initial volume of free air

$$-\Delta V = (1-S)V_V$$

$$-\frac{\Delta V}{V} = (1-S)V_V/V = (1-S)n_0$$

The pore pressure change required to give saturation can thus be calculated as

$$\frac{\Delta p}{p_0} = \frac{n_0(1-S)}{-n_0(1-S) + n_0(1-S+SH)}$$

$$\therefore \Delta p_S = p_0 \left\{\frac{1-S}{SH}\right\} \quad \text{— b.}$$

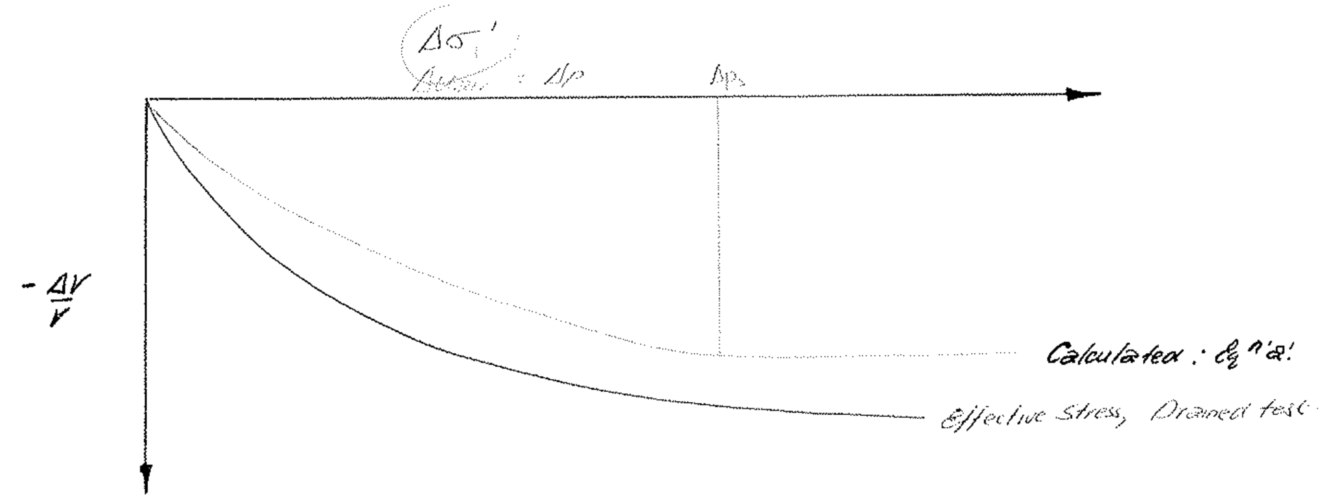
Note that at high altitude p_0 is smaller and thus Δp_S will be smaller.

<u>The best method for solving this problem is graphically</u>

1. From the drained test the volume change effective stress relationship can be obtained.

2. From Equation 'a' the volume change Δu_{air} relationship can be obtained

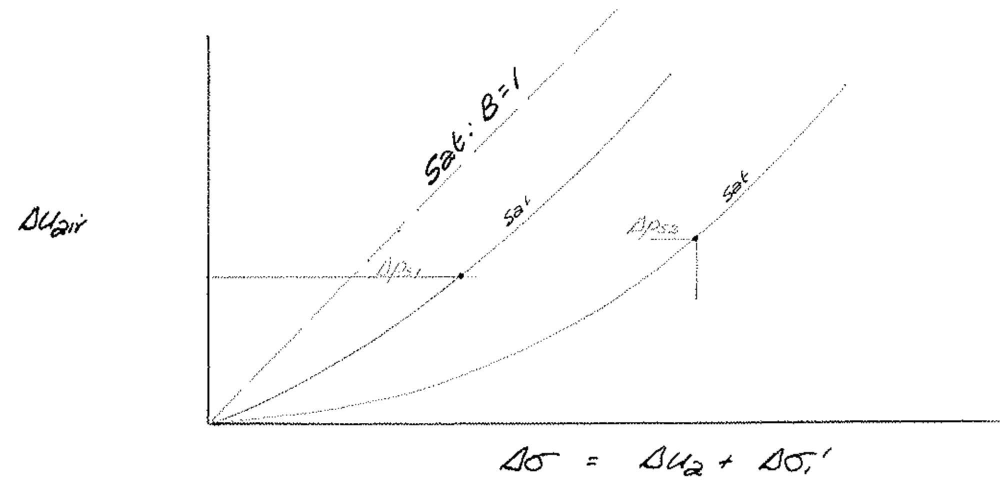
Thus there are two curves, as follows.



Thus for a series of volume changes one can obtain

$$\Delta\sigma = \Delta\sigma' + \Delta u_{air}$$

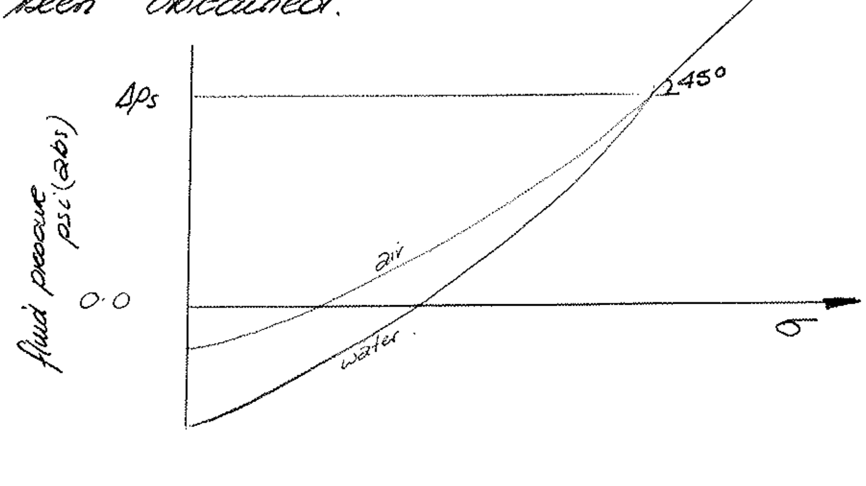
and hence plot a graph of Δu_{air} vs $\Delta\sigma$.



Note: Strictly $\Delta\sigma = \Delta\sigma' + \Delta u_a - \Delta\{\chi(u_a - u_w)\}$ and although the end term is neglected in the upper graph, for a real soil with capillary forces the term $\Delta\chi(u_a - u_w)$ cannot be ignored.

At low moisture contents the water may have very high tensile stresses.

From experiment the following type of results have been obtained.



Note: The curves converge due to the fact that the air bubbles become larger and not smaller when the sample is compressed.

Effect of measuring air pressure.

Field equipment usually measures air pressure. This will be on the safe side since $u_{air} > u_{water}$ and hence the calculated effective stresses will be lower.

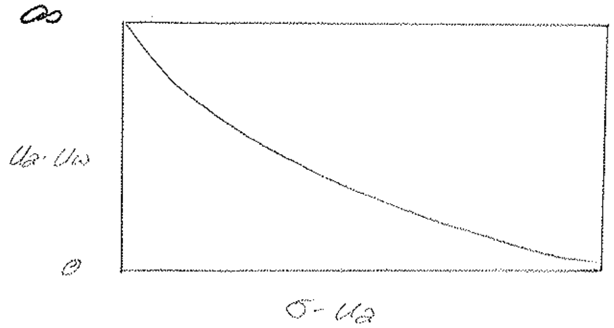
Relationship between u_{air} & u_{water}.

$$u_a = u_{a0} + \Delta u_a \qquad u_w = u_{w0} + \Delta u_{w0}$$

$$\text{Now} \quad B_a = \frac{\Delta u_a}{\Delta \sigma} \qquad B_w = \frac{\Delta u_w}{\Delta \sigma}$$

$$\therefore \quad u_a = u_{a0} + B_a \Delta\sigma \qquad u_w = u_{w0} + B_w \Delta\sigma.$$

Now $u_a - u_w$ is not independant of stress. If u_a is measured then an idea of u_w can be obtained from a graph as follows.



In general

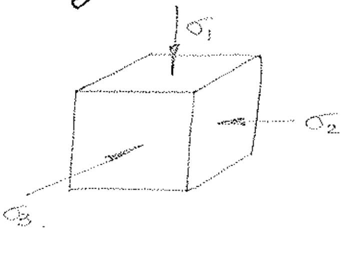
$$\frac{u_a - u_w}{\sigma - u_a} = 0 \rightarrow 1.0$$

The difference between u_a & u_w may be as large as $\sigma - u_a$ and is hence significant.

Pore Pressure changes due to deformation in General.

In general a change in pore pressure results from any change in shear stress

Consider soil as being elastic and isotropic in all directions. Consider pores as filled with water. Consider that the sample is subjected to σ_1 & σ_2 & σ_3 and then undergoes a change in stresses $\Delta\sigma_1, \Delta\sigma_2, \Delta\sigma_3$.



Let the resulting change in pore pressure $= \Delta u$.

$$\left.\begin{aligned} \Delta\sigma_1' &= \Delta\sigma_1 - \Delta u \\ \Delta\sigma_2' &= \Delta\sigma - \Delta u \\ \Delta\sigma_3' &= \Delta\sigma - \Delta u. \end{aligned}\right\} \quad \text{—} \; (1)$$

$e_1 \;\; e_2 \;\; e_3$: are corresponding strains - +ve = compression

E : Youngs Modulus

μ : Poisson's ratio

<u>The strains are thus</u>

$$\left.\begin{aligned} e_1 &= \frac{1}{E}\left\{\Delta\sigma_1' - \mu(\Delta\sigma_2' + \Delta\sigma_3')\right\} \\ e_2 &= \frac{1}{E}\left\{\Delta\sigma_2' - \mu(\Delta\sigma_1' + \Delta\sigma_3')\right\} \\ e_3 &= \frac{1}{E}\left\{\Delta\sigma_3' - \mu(\Delta\sigma_1' + \Delta\sigma_2')\right\} \end{aligned}\right\} \quad \text{—} \; (2)$$

Thus $\quad e_1 + e_2 + e_3 = \frac{1}{E}\left\{\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3'\right\}\left\{1 - 2\mu\right\} \quad$ — (3)

Now for small strains $\quad e_1 + e_2 + e_3 = -\Delta V/V \quad$ — (4)

$$\therefore \quad -\frac{\Delta V}{V} = \frac{1}{E}\left\{\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3'\right\}\left\{1 - 2\mu\right\} \quad \text{—} \; (5)$$

Now as before for an equal all round change in effective stress we have

$$-\frac{\Delta V}{V} = C_c\,\Delta\sigma'$$

ie from (5) $\quad -\Delta V/V = \frac{3}{E}(1-2\mu)\,\Delta\sigma'$

$$\therefore \quad C_c = \frac{3}{E}(1-2\mu) \quad \text{—} \; (6)$$

Thus we can rewrite 5.

$$\underline{-\frac{\Delta V}{V} = \frac{C_c}{3}\left\{\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3'\right\}}$$

Equation (1) gives

$$\underline{\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3' = \Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3 - 3\Delta u.}$$

Thus the volume change in terms of total stress is

$$\underline{\underline{-\frac{\Delta V}{V} = \frac{C_c}{3}\left\{\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3 - 3\Delta u\right\}}}$$

Now the change volume of the pore water is

$$-\Delta V = nV \cdot C_w \cdot \Delta u.$$

Again, as before, since there is no drainage we have that these volume changes must be equal.

$$\therefore \quad nV \cdot C_w \cdot \Delta u = V \frac{C_c}{3} \left\{ \Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3 - 3\Delta u \right\} \qquad (13)$$

$$\therefore \quad n \cdot \frac{C_w}{C_c} \cdot \Delta u + \Delta u = \tfrac{1}{3} \left\{ \Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3 \right\}.$$

Thus

$$\Delta u = \left\{ \frac{1}{1 + n \frac{C_w}{C_c}} \right\} \tfrac{1}{3} \left(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3 \right)$$

This is virtually the same equation as for equal all round stress.

$$\therefore \quad \Delta u = B \cdot \tfrac{1}{3} \left(\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3 \right)$$

NB: B applies to a <u>change</u> in <u>total</u> stress

<u>Pore pressure changes in an undrained triaxial test.</u>

In this case $\Delta\sigma_2 = \Delta\sigma_3$

$$\therefore \quad \Delta u = B \cdot \tfrac{1}{3} \left(\Delta\sigma_1 + 2\Delta\sigma_3 \right)$$

Separating out the deviator stress

$$\Delta u = \frac{B}{3} \left\{ (\Delta\sigma_1 - \Delta\sigma_3) + 3\Delta\sigma_3 \right\}$$

$$\therefore \quad \Delta u = B \left\{ \frac{(\Delta\sigma_1 - \Delta\sigma_3)}{3} + \Delta\sigma_3 \right\}$$

This is the equation that would apply to a perfectally elastic and isotropic soil. However for actual soils the equation

$$\Delta u = B \left\{ \Delta\sigma_3 + A (\Delta\sigma_1 - \Delta\sigma_3) \right\} \quad \text{applies.}$$

where A depends on the soil type and the stress history (N.C. or O.C) and also on the extent to which failure is approached.

In many field problems $\Delta\sigma_1$ is known but $\Delta\sigma_3$ is not really "well known". The major principle stress is the more important variable but to what extent.

$$\Delta u = B\{\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)\}$$

$$= B\{\Delta\sigma_1 - (1-A)(\Delta\sigma_1 - \Delta\sigma_3)\}$$

we define $\frac{\Delta u}{\Delta\sigma_1} = \bar{B}$

Hence $\bar{B} = B\left\{1 - (1-A)\frac{(\Delta\sigma_1 - \Delta\sigma_3)}{\Delta\sigma_1}\right\}$

$$= B\left\{1 - (1-A)\left(1 - \frac{\Delta\sigma_3}{\Delta\sigma_1}\right)\right\}$$

For a normally consolidated soil $A \to 1.0$ and thus

$$\bar{B} \to B \to 1.0$$

ie completely insensitive to $\frac{\Delta\sigma_3}{\Delta\sigma_1}$

<u>Field Case - Plane Strain</u>

In the field we may have no yeild in the third direction:

$$e_2 = 0$$

$$\therefore \quad 0 = \frac{1}{E}\{\Delta\sigma_2' - \mu(\Delta\sigma_1' + \Delta\sigma_3')\}$$

$$\therefore \quad \Delta\sigma_2' = \mu(\Delta\sigma_1' + \Delta\sigma_3')$$

Thus $-\frac{\Delta V}{V} = \frac{1}{E}\{\Delta\sigma_1' + \mu(\Delta\sigma_1' + \Delta\sigma_3') + \Delta\sigma_3'\}\{1-2\mu\}$

Now $\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3' = \Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3 - 3\Delta u$. as before

or $\Delta\sigma_1' + \Delta\sigma_3' = \Delta\sigma_1 + \Delta\sigma_3 - 2\Delta u$

$$\therefore \quad -\frac{\Delta V}{V} = \frac{1}{E}\{(\Delta\sigma_1' + \Delta\sigma_3')(1+\mu)(1-2\mu)\}$$

$$= 1/E\{(\Delta\sigma_1 + \Delta\sigma_3 - 2\Delta u)(1+\mu)(1-2\mu)\}$$

We define $C_p = \frac{2}{E}(1-2\mu)(1+\mu)$

$$\therefore \quad -\Delta V = V\frac{C_p}{2}\{\Delta\sigma_1 + \Delta\sigma_3 - 2\Delta u\}$$

Thus as before:

$$nV \cdot C_w \cdot \Delta u = V\frac{C_p}{2}\{\Delta\sigma_1 + \Delta\sigma_3 - 2\Delta u\}$$

$$\therefore \; n \frac{C_\omega}{C_p} \Delta u + \Delta u = \frac{(\Delta\sigma_1 + \Delta\sigma_3)}{2}$$

Thus $$\Delta u = \frac{1}{1 + n \frac{C_\omega}{C_p}} \; \frac{\Delta\sigma_1 + \Delta\sigma_3}{2}.$$

Again the equation is in the <u>same form as before</u>

where $$C_p = \frac{2(1-2\mu)(1+\mu)}{E}$$

before $$C_c = \frac{3}{E}(1-2\mu)$$

<u>Thus for plane strain:</u> $$B_p = \frac{1}{1 + n\, C_\omega / C_p}.$$

$$\Delta u = \frac{B_p}{2}\left\{2\Delta\sigma_3 + (\Delta\sigma_1 - \Delta\sigma_3)\right\}.$$

$$= B_p \left\{\Delta\sigma_3 + \frac{\Delta\sigma_1 - \Delta\sigma_3}{2}\right\}.$$

Thus from purely elastic considerations the pore pressures in the field should be higher. because the pore pressures due to the deviatoric stress are higher.

<u>Meaning of the term C_p.</u>

In plane strain if $\Delta\sigma_1' = \cancel{\Delta\sigma_2}' = \Delta\sigma_3'$

then
$$-\Delta V/V = \frac{(1-2\mu)(1+\mu)}{E} \times 2\Delta\sigma_1'$$

$$= C_p \text{ (as defined)} \cdot \Delta\sigma'$$

Thus C_p is the compressibility in plane strain for equal values of $\Delta\sigma_1'$ & $\Delta\sigma_3'$.

NB: The term compressibility should usually be used only for equal all round pressure.

Field Equation:

$$\Delta u = B_p\left\{\Delta\sigma_3 + A_p(\Delta\sigma_1 - \Delta\sigma_3)\right\}$$

Note: The values of A_p & B_p are normally assumed to be the values measured in the triaxial test.

Typical A values. : Remoulded Weald Clay.

	$\dfrac{\Delta\sigma_1 - \Delta\sigma_3}{(\Delta\sigma_1 - \Delta\sigma_3)_f} = 1/3$	1/2	1·0
N.C. Clay.	+0·75	+0·67	+0·92
O.C = 4	+0·27	+0·18	+0·03 Delete
O.C = 8	+0·18	+0·08	−0·25.

Deformation under conditions of zero lateral yield

Under these conditions we must have $e_2 = e_3 = 0$
also $\Delta\sigma_2' = \Delta\sigma_3'$

Thus

$$e_2 = e_3 = 0 = \frac{1}{E}\left\{\Delta\sigma_3' - \mu(\Delta\sigma_1' + \Delta\sigma_3')\right\}$$

$$\therefore \quad \mu\Delta\sigma_1' = \Delta\sigma_3'(1-\mu)$$

$$\therefore \quad \Delta\sigma_3' = \left(\frac{\mu}{1-\mu}\right)\Delta\sigma_1'$$

Ko : Coefficient

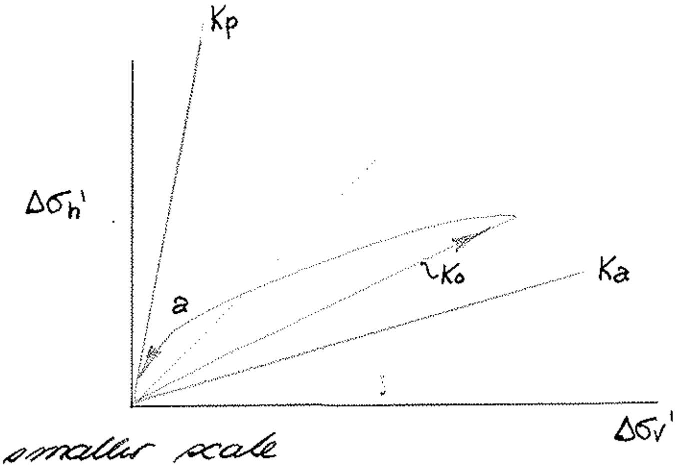
Now the ratio $\dfrac{\Delta\sigma_3'}{\Delta\sigma_1'} = K_o$ the coefficient of earth pressure at rest.

Thus for elastic soils $K_o = \dfrac{\mu}{1-\mu}$

but in practice K_o depends on the stress history and the nature of the material − particularly ϕ'.

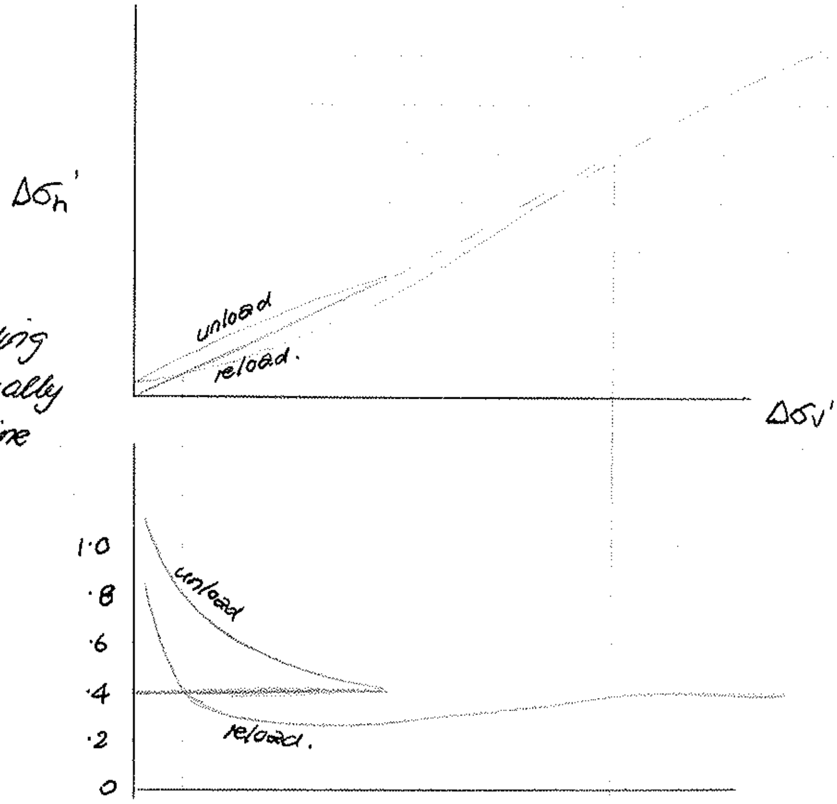
Consider the loading and unloading of a typical soil.

Because of the shape of the unloading curve it is possible for an overconsolidated clay to have a K_o value greater than 1.0.



Considering this cycle on a smaller scale it can be shown that after the first loading and unloading cycle lateral pressure does not build up but rather tends to fall away.

On first loading one gets virtually a straight line



K_0 must always be slightly above K_a and as an empirical rule for Normally Consolidated clays

$$K_0 = 1 - \sin\phi.$$

since $K_a = \dfrac{1-\sin\phi'}{1+\sin\phi'}$

we have $\dfrac{K_0}{K_a} \doteqdot 1 + \sin\phi'$ ie $K_0 > K_a$.

<u>Volume changes in the oedometer test.</u>

As before
$$-\Delta V/V = e_1 + e_2 + e_3$$
$$= \frac{C_c}{3}\left\{\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3'\right\}.$$

But $\Delta\sigma_3' = \Delta\sigma_2' = K_0\,\Delta\sigma_1'$

Thus
$$-\frac{\Delta V}{V} = \frac{C_c}{3}\left\{\Delta\sigma_1' + 2K_0\,\Delta\sigma_1'\right\}$$
$$= \Delta\sigma_1'\ \frac{C_c}{3}\left(1 + 2K_0\right).$$

Now the compressibility in the oedometer is generally denoted by the term m_v

where

$$-\frac{\Delta V}{V} = m_v\,\Delta\sigma_1'$$

Hence $\quad m_v = \dfrac{(1+2K_0)}{3} C_c$

If $K_0 = 0.7$ then $m_v = 0.80\, C_c$ for elastic soils.

This difference between m_v & C_c affects the prediction of pore pressures

However actual soils depart somewhat from this elastic theory

This departure is best seen by separating the volume change due to all round pressure change from that due to deviator stress

In general $\quad -\dfrac{\Delta V}{V} = \dfrac{C_c}{3}\left\{\Delta\sigma_1' + \Delta\sigma_2' + \Delta\sigma_3'\right\}$

In oedometer $\quad -\dfrac{\Delta V}{V} = \dfrac{C_c}{3}\left\{3\Delta\sigma_3' + (\Delta\sigma_1' - \Delta\sigma_3')\right\}$

$$= \dfrac{C_c}{\cancel{3}}\left\{\Delta\sigma_3' + \dfrac{\Delta\sigma_1' - \Delta\sigma_3'}{3}\right\}$$

which is an equation similar in form to the earlier one for Δu i.t.o. total stress.

For an actual soil we have

$$-\dfrac{\Delta V}{V} = C_c\left\{\Delta\sigma_3' + S_d(\Delta\sigma_1' - \Delta\sigma_3')\right\}$$

where S_d = The structural parameter.
= f(soil type, stress history etc)
$\doteqdot$ 1.0 for N.C. clays.
$\doteqdot$ 1/3 for H.O.C. clays.

Thus for the oedometer one can write

$$-\dfrac{\Delta V}{V} = C_c\left\{K_0\Delta\sigma_1' + S_d(\Delta\sigma_1' - K_0\Delta\sigma_1')\right\}$$

$$= C_c \cdot \Delta\sigma_1'\left(K_0 + S_d(1 - K_0)\right)$$

$$= C_c \cdot \Delta\sigma_1' \cdot S_d\left\{1 + K_0\left(\tfrac{1}{S_d} - 1\right)\right\}$$

Thus $\quad m_v = S_d\left\{1 + K_0\left(\dfrac{1}{S_d} - 1\right)\right\} C_c$

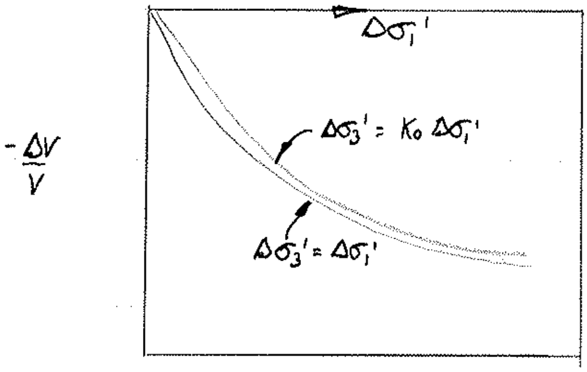
For a fairly loose sand $\quad K_0 = 0.4 \quad S_d = 0.33.$

$\therefore \quad m_v = 0.6\, C_c$

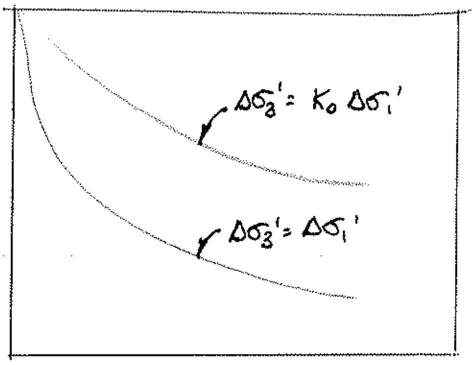
" " plastic clay $\quad m_v = 0.94\, C_c$

Thus oedometer test on clay gives a compressibility value

close to the true all round pressure value.
For sands and for O.C. clays however the oedometer does not measure very close to the true compressibility
The oedometer is the correct one as in the field one does not really get all round pressure.



N.C. Clay.

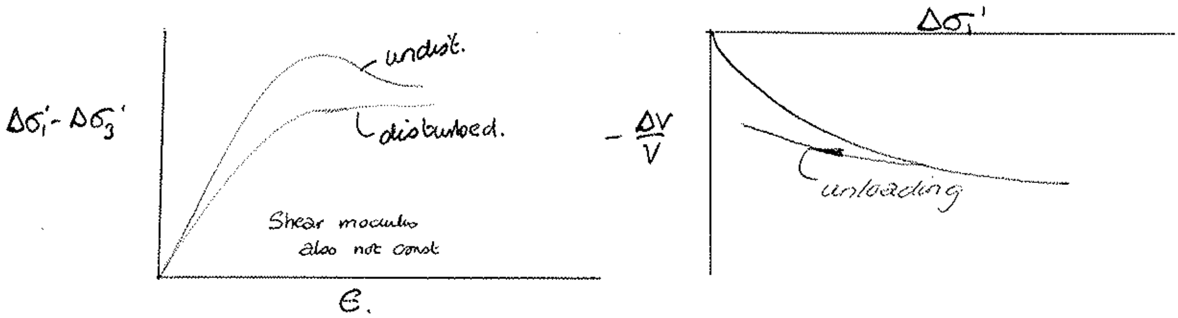


Sands & O.C. Clays.

In compacted fills we may be in quite large error when we obtain $\bar{B}$ from the triaxial. May considerably overestimate p.w.p because triaxial gives higher compressibility.

Difficulties from using elastic theory in practice.

1. The deformation modulus is not constant.



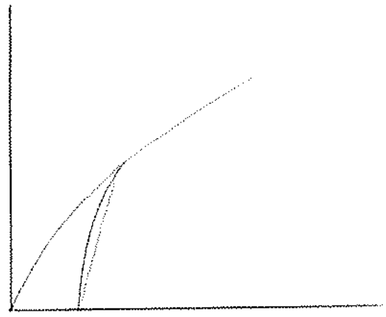
The modulus varies with magnitude of stress ; thus with depth. The shallower, the lower the stress and the greater the volume compressibility

2. The deformation modulus varies according to whether the stress is increasing or decreasing

For N.C. clays the ratio : $\frac{\text{Loading modulus}}{\text{Unloading modulus}} \approx$ upto 10

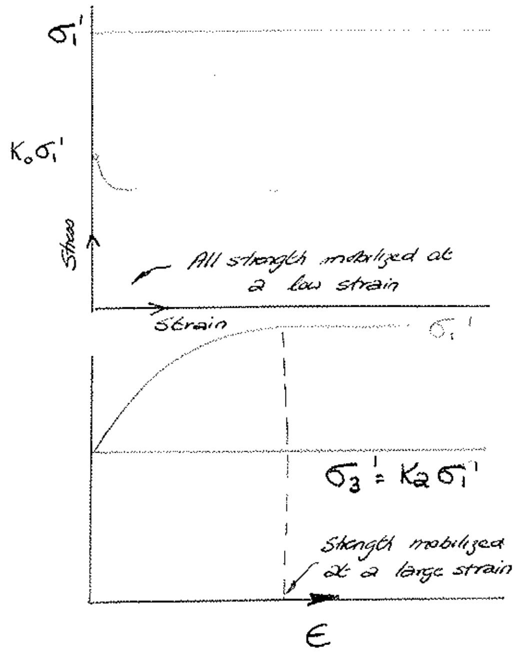
There is a similar effect when one considers the shear

deformations
Thus for heave of foundations or yeild of retaining walls a test with a stress decrease should be used to obtain the appropriate modulus.



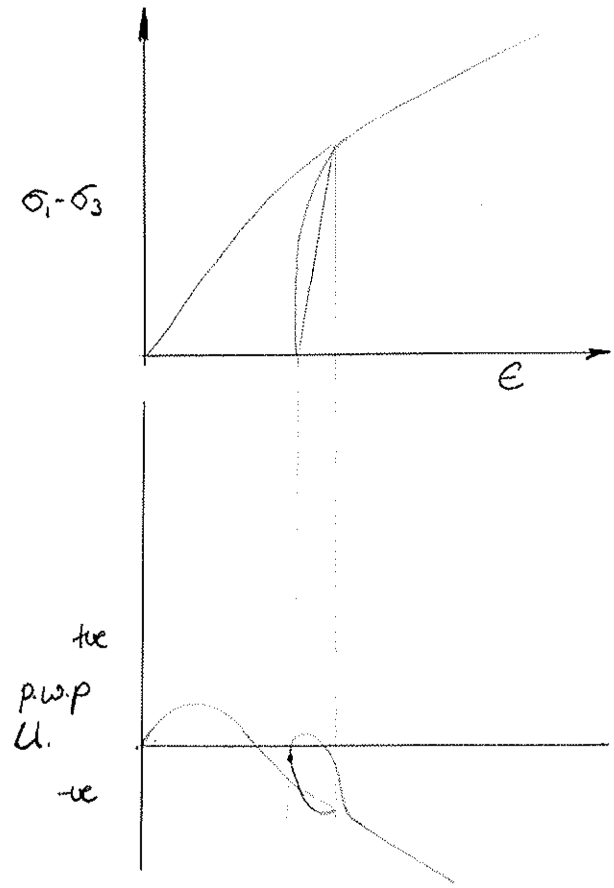
eg Retaining wall.
The proper test would be one in which consolidation was done under K_0 conditions and then failure was obtained by decreasing $\Delta\sigma_3'$ and keeping $\Delta\sigma_1'$ constant.

The usual drained test is to have σ_3' = constant and to increase σ_1' until failure is reached. Consolidation having been allowed under an all round pressure



3. <u>The effect of the principle of superposition w.r.t p.w.p.</u>

The load may be removed but there remains a built in negative pore pressure.
Because of these effects of loading history one cannot really apply the values in the field where the stress history is different.



Irreversibility of pore water changes - implications w.r.t undisturbed sampling

On sampling the stresses in the ground are released. When one puts the stresses back on again one does not get the field pore-pressures reproduced.

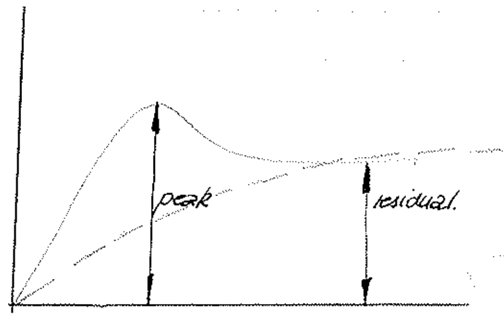
It is difficult to predict field behaviour according to lab. results and it must be noted that the reaction of the in-situ sample to changes in stress are different to those of the lab sample.

The field e-log p curve is different to the lab.

Undrained tests are also affected. It is perhaps better to stress above the ground pressure so as to cancell out the built in pore pressure.

MEASUREMENT OF SHEAR STRENGTH

The shear strength of a soil is defined as the maximum shear stress that soil can withstand under a specified set of conditions.



Usually interested in the peak value but residual must also be considered.

In a very compressible soil huge deformations are needed to give peak values

In this case it is easier to get peak values if the soil is tested

a. Active failure ie σ_1' constant and decrease σ_3'

b. Undrained test. This is at constant volume and there is no ~~volume change~~ compressibility effect.

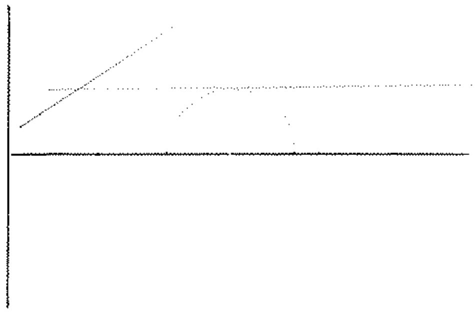
Conditions which controll the shear strength.

a. Normal pressure on the shear plane.

b: Conditions of drainage

If the excess pore pressures are not allowed to dissipate then the effective stresses will be less than the applied stresses.

If an undrained test is performed on a saturated material then because $B = 1{\cdot}0$ the effective stresses are independant of σ_3. Thus all the failure circles are the same size and if pore pressures are measured only one effective stress circle is obtained



3. The rate of strain

This has two effects

i/ A certain part of the measured strength is due to a viscous component of strength and this is particularly marked under triaxial loads.

ii/ Even if there is provision for drainage enough time must be allowed for the water to dissipate. This time might be quite large for low permeability soils

4. Orientation of sample - if undisturbed.

The sample must be taken so that the shear plane (at $45 + \phi'/2$) is in the correct direction

eg in L.C. a 50° oriented sample had only 80% of the vertical sample strength.

In specifying tests they may belong to either of the following classes.

a. Undrained Tests.

1. Zero drainage during application of the normal stress
2. Zero drainage during shear.

If p.w.p measured then properties i.t.o effective stress - otherwise in terms of total stress.

b: Consolidated Undrained Tests

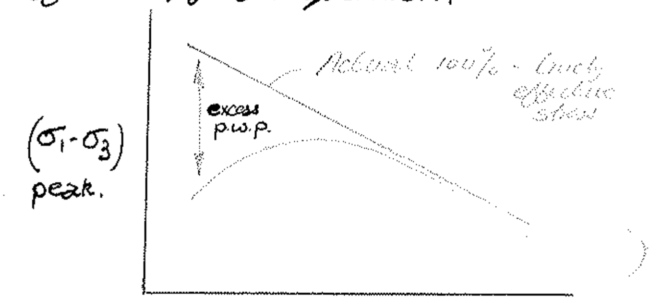
1. Full drainage (consolidation) under the applied normal stress.
2. Zero drainage during shear - with a resultant +ve or -ve excess porewater pressure being set-up.

c. Fully drained tests.

1. Full consolidation allowed under the applied normal stress.
2. Full dissipation of p.w.p allowed during shear.

For practical purposes the rate of shearing is usually worked out on the basis of 95% dissipation.

Since there are negligable pore pressures at failure the results are interms of effective stresses.



The time required for the test depends on

1/ Coefficient of consolidation - hence k.
2/ Square of the drainage path.

The rate of consolidation can be measured during the first stage of the test. One uses this together with an estimate of the failure strain to calculate the required testing time

d. Constant water content test. (partly sat. soils).

Air is allowed to drain from the voids during the consolidation stage or both during consolidation and shear.

No water drains since Uw is below atmospheric (or less than the back pressure) and is measured. A high back pressure is usually used to enable the pore pressure to be measured without cavitation

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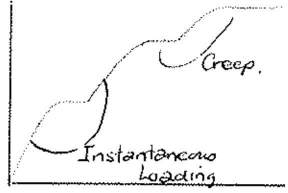
LOADING

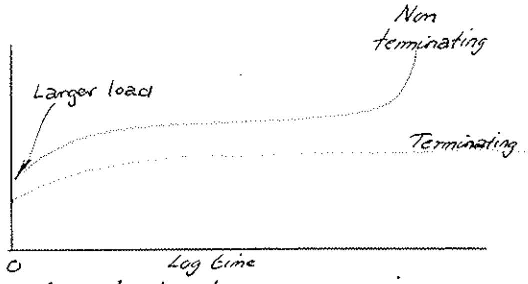
a: Incremental loading

A series of increment in stress are made. The strain and the amount of creep is noted under each increment.

There are three difficulties

i. Creep will continue for a long time under each load and it is difficult to recognise when non-terminating creep has begun.





At a time t after loading the curve shapes for terminating and non-terminating creep are the same. Thus one cannot tell at what stage one is.

ii. The dead load required to cause failure is not known in advance and hence the rate of loading to give a specified time to failure is not known. Thus the viscous component is uncertain and is different from sample to sample.

iii. The peak strength is the limit and the behaviour after peak cannot be investigated.

b: Controlled rate of strain testing.

Deformation is applied at a controlled rate and the corresponding stresses are measured.

1. The peak is clearly defined and the problem of terminating and non-terminating creep does not occur

2. The strain required to produce failure is more accurately predictable and thus the time to failure can be controlled. This is important in drained tests. It is also important in undrained tests both for pore pressure equalization throughout the sample and to controll the viscous component of strength.

3. The behaviour of the soil after peak can be investigated. In very rigid soils the energy stored in the proving ring & the tie rods may be sufficient to cause a jump in the loading as one passes peak. Thus for rocks a rigid load cell & testing rig is used.

The Direct Shear Box.

The form most commonly used today is a test in single shear with a controlled rate of strain

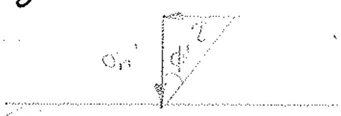
The shear box is suitable for drained tests - having a short drainage path.

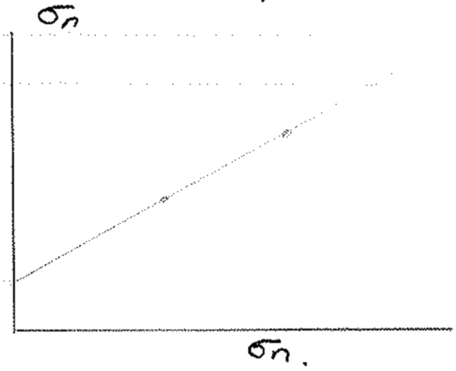
However the distribution of stress is somewhat unknown and the method of analysis of results is somewhat controversial.

a: <u>Conventional method.</u>

$$\frac{\tau}{\sigma_n} = \tan \phi'$$

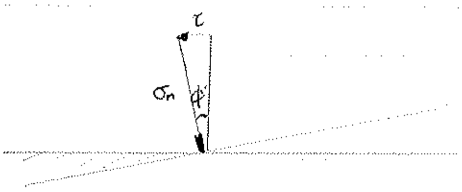
Thus the results are plotted directly as shown.

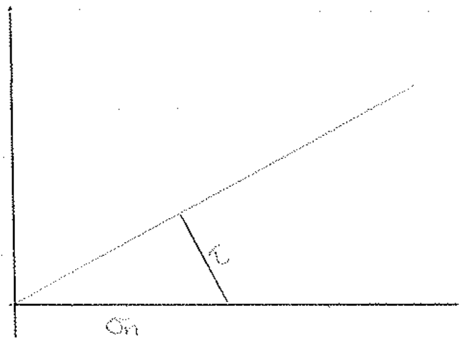




b. <u>Hill method.</u>

$\tau / \sigma_n' = \sin \phi'$





The difference can be fairly large for large values of ϕ'

$\phi_c = 30°$ $\quad \phi_{hill} = 35°$

$\phi_c = 20°$ $\quad \phi_{hill} = 21{\cdot}3°$.

The shear box can thus be used for clays without too much uncertainity

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<u>The Conventional Triaxial Test.</u>

The types of loading which can be followed are

1. <u>Compression Test.</u>
   1. σ_{axial} increased σ_{radial} constant.
   2. σ_a const – σ_r decreased.
   3. Average principal stress constant σ_a increased – σ_r decreased.
2. <u>Extension Test</u>
   1. σ_a decreased – σ_r constant
   2. σ_a const – σ_r increased
   3. Average principal stress const. σ_a decreased and σ_r increased

An <u>active failure</u> is when σ_3 decreases and σ_1 remains constant.

A <u>Passive failure</u>, σ_1 increases and σ_3 remains the same.

The extension test has certain difficulties

1. Mechanically tricky

2. There is the possibility of necking of the sample, as the area decreases on extension. Peak not defined.

3. The failure plane in compression is at $45 + \phi/2$ and thus covers the whole sample. making it reasonable to take the whole X sectional area. In the extension test the plane is at $45 - \phi/2$ and thus lies in the area of reduced cross section.

For an accurate determination of $(\sigma_1 - \sigma_3)$ in extension tests the stress - strain curve must be calculated as the test proceeds. As soon as peak is approximatly reached the test must be stopped and the cross sectional area measured.

The effect of end friction

Owing to end friction strain is not quite uniform but tests have shown that this does not make any significant difference to the strengths provided $H/D > 2$

The degree of end friction does affect the volume change and the strains but apparently the max rate of volume change is unaffected.

Uses of the Triaxial Test.

The uses are

1/ To relate stress, deformation and volume change

2/ To controll the drainage to give either drained or undrained conditions

There are problems with regard to the redistribution of porewater due to the unequal strains set up.

The test can also be used to measure

3/ Coefficient of earth pressure at rest

4/ Consolidation characteristics

The principal disadvantages are

1/ The intermediate principal stress cannot be varied to simulate plane strain conditions

2. The directions of the principal stresses cannot be progressively changed.

3. End restraint may modify the various

relationships between stress, strain and pore pressure. and volume change.

Practical Failure Criteria

Resistance to shear at low stresses is largely controlled by the forces between particles and can in general be separated into two terms.

i: Frictional term - being approximately proportional to effective stress.

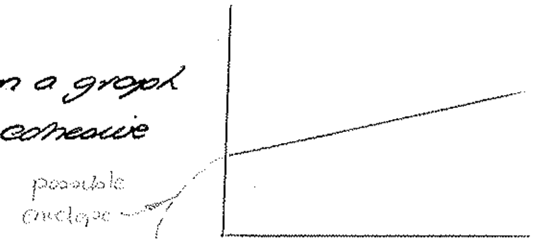
ii. Cohesion term - which depends on water content, state of packing and cementing agents.

Modified Coulomb failure criteria.

$$\tau_f = c' + (\sigma_n - u) \tan\phi'$$
$$= c' + \sigma_n' \tan\phi'$$

where c' = cohesion intercept. } in terms of
ϕ' = angle of shearing resistance } effective stress.

Note: c' is essentially a point on a graph and is not necessarily a cohesive force.

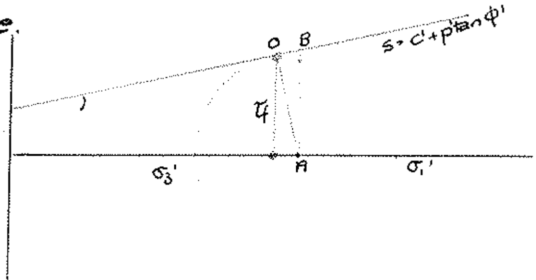


Variability in c' and ϕ'

For ranges in stress which do not involve large changes in water content (or volume) c' & ϕ' are almost independant of the type of test used to measure them.

The main differenle between tests lies in the values of excess pore pressure at failure.

Failure criteria on Mohr envelope.



In general the criteria may be represented by a straight line from triangle OAB.

$$\cos\phi' = \frac{AO}{AB} = \frac{(\sigma_1' - \sigma_3')/2}{c' + \frac{\sigma_1' + \sigma_3'}{2} \tan\phi'}$$

$$\therefore \quad \frac{(\sigma_1 - \sigma_3)_f}{2} = c' \cos\phi' + \frac{(\sigma_1' + \sigma_3')_f}{2} \tan\phi' \cos\phi'$$

$$\therefore \quad \frac{(\sigma_1 - \sigma_3)_f}{2} = c' \cos\phi' + \frac{(\sigma_1' - \sigma_3' + 2\sigma_3')_f}{2} \sin\phi'$$

$$\therefore \quad \left(\frac{\sigma_1 - \sigma_3}{2}\right)_f = \frac{c' \cos \phi' + (\sigma_3 - u) \sin \phi'}{1 - \sin \phi'}$$

This equation thus represents coulombs failure criteria in terms of principal stresses where

$(\sigma_1 - \sigma_3)$ = Deviator stress.

$(\sigma_3 - u)$ = σ_3' = minor effective principal stress at failure.

Undrained strength of saturated soils

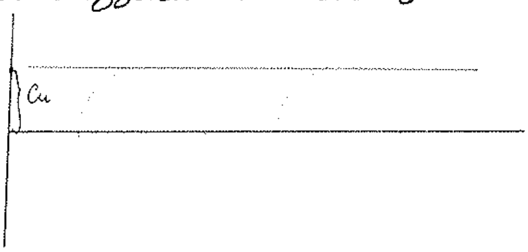
A series of samples of clay are consolidated under an all round pressure $= p$

With no drainage the ambient pressure increased by $\Delta\sigma_3$ and axial stress by $\Delta\sigma_1$ until failure occurs.

Principal total stresses at failure

$$\sigma_1 = p + \Delta\sigma_1$$
$$\sigma_3 = p + \Delta\sigma_3$$

The failure state can be represented by a Mohr stress circle. If the other samples are tested with different values of $\Delta\sigma_3$ it is found that the deviator stress at failure is entirely independant of the magnitude of $\Delta\sigma_3$



Thus saturated sands and clays on the whole behave with respect to total stresses, as purely cohesive materials with

$$\phi_u = 0$$
$$c_u = \tfrac{1}{2}(\sigma_1 - \sigma_3)_f$$

stress at failure independ.

The increase $\Delta\sigma_3$ is reflected by an equal increase Δu and thus effective

The same results apply equally to a set of undisturbed samples taken from the ground, all of which have the same capillary pressure. In this case the value of p is not in general equal to the average in situ effective stress.

The A value measured in a test on a sample of natural ground is very different from the A value measured in situ under a similar change in shear stress. This results from the stress history given to the sample by changes in pore pressure which occur during sampling and preparation, due to the removal of the in-situ stresses, quite apart from mechanical disturbance due to the sampler itself. The release of the deviator stress on the insitu sample N.C. with no lateral yeild is a major factor contributing to this effect. Tests have shown that the effective stress in the sample when under an all round pressure or unconfined can be $<\frac{1}{2}$ effective in-situ overburden pressure.

Yet tests to failure give values which satisfy field values. This is consistent with the obs. that for a limited range of soil types and strength paths strength and water content are uniquely related.

If this is the reason why undrained compression tests give the correct data for $\phi = 0$ analysis then it wrong to reconsolidate the samples in the laboratory under the existing overburden pressure. This will inevitabley lead to an overestimate of the insitu strength.

<u>In the lab.</u>

$$\Delta u_f = \Delta\sigma_3 + A_L(\Delta\sigma_1 - \Delta\sigma_3)$$

<u>In the field</u> $\Delta u_f = \Delta\sigma_3 + A_f(\Delta\sigma_1 - \Delta\sigma_3)$.

where $A_{lab} \neq A_{field}$

thus the effective stresses in the lab are not equal to those in the field. For N.C. clays $A_{lab} > A_{field}$.

$\therefore$ Δu lab $> \Delta u$ field $\therefore$ lab effective stresses $<$ field.

Stiff fissured clays $\phi_u = 0$ for all ambient pressures greater than a value approximately equal to the overburden pressure acting on the sample in-situ. At lower ambient pressures the strength is less due to failure occuring along the fissures.

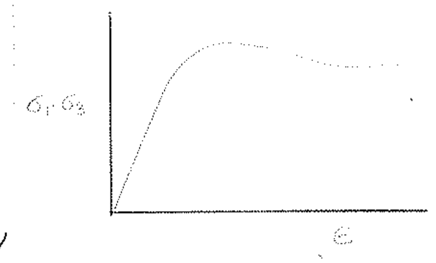
Materials with a marked tendancy to dilate have negative pore pressures during shear and the pore-pressure may drop below -1 atmosphere before failure is reached. Cavitation thus results in no further decrease in pore pressure. Thus plotted in terms of total stress the strength increases with increasing values of $\Delta\sigma_3$, for $B \neq 1.0$ at failure. For values of σ_3 greater than that required to prevent cavitation $B = 1.0$ and $\phi_u = 0$.

Analysis of the Undrained test on saturated soils

For $B = 1.0$ we write

$$\Delta u = \Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)$$

Two stages in performing the test



a. During application of all round increase in stress $\Delta\sigma_3$ an equal pore pressure is set up ($B=1$) and hence the effective stresses are unaffected $\therefore$ strength indep. of $\Delta\sigma_3$

b. Activation of the stress difference $\Delta\sigma_1 - \Delta\sigma_3$ sets up an additional pore pressure of Δu_{df}.
Hence the total pore pressure change Δu_f is

$$\Delta u_f = B\{\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)_f\}$$
$$\Delta u_f = \Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)$$
$$\Delta u_{df} = A(\Delta\sigma_1 - \Delta\sigma_3)$$

Effective stresses at failure

$$\sigma_1' = p + \Delta\sigma_1 - \{\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)\}$$
$$= p + (1-A)\{\Delta\sigma_1 - \Delta\sigma_3\}_f$$

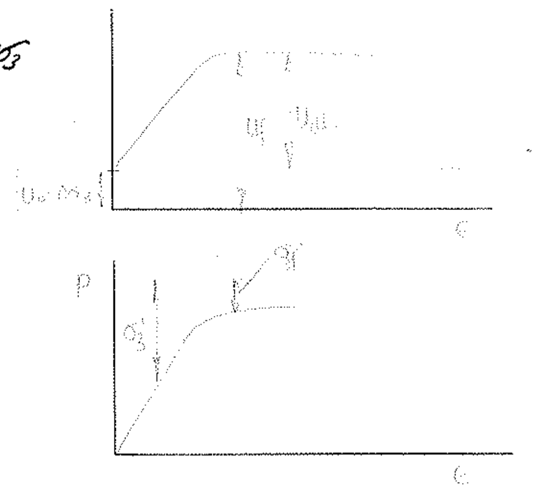
$$\sigma_3' = p + \Delta\sigma_3 - \{\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)_f\}$$
$$= p - A(\Delta\sigma_1 - \Delta\sigma_3)$$

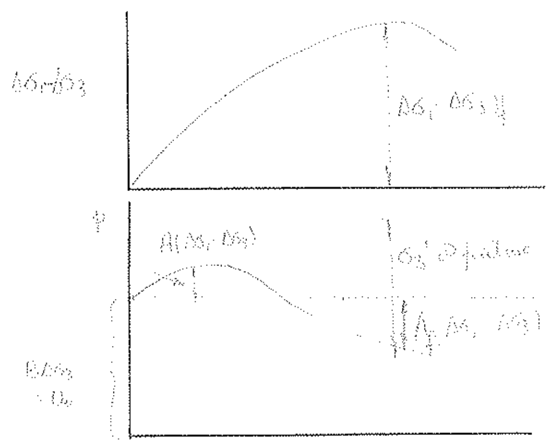
Normally consolidated soils $A_f \doteq 1.0$

$\sigma_1' = p$ ie no change.

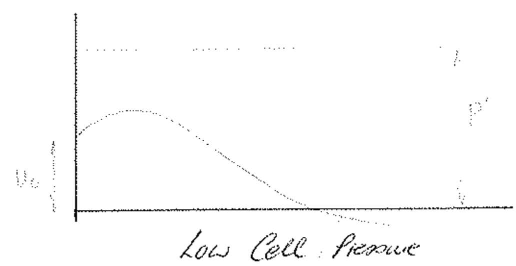
$\sigma_3' = p - (\Delta\sigma_1 - \Delta\sigma_3)_f$

ie stress drop.





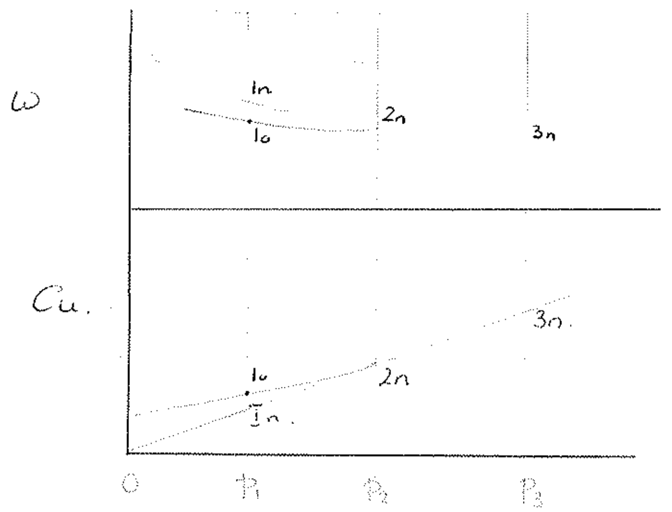
High Cell Pressure

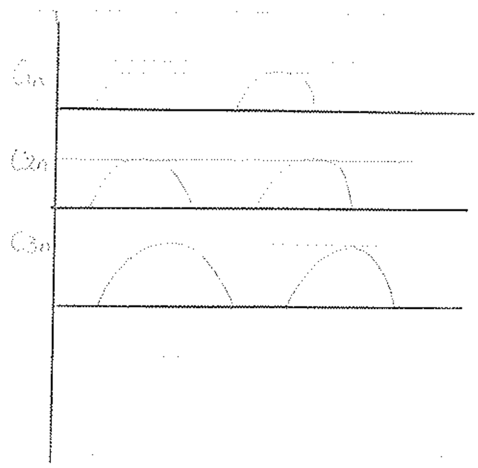


Low Cell Pressure

Negative pwp at failure if $\Delta\sigma_3$ low

The effect of varying the consolidation pressure.





Clay consolidated from a slurry under a pressure p_1.

Water content will be I_n @ p_1

A series of undrained tests yeilds $c_u = C_{In}$

When plotted under a larger consolidation pressure p_2 we have C_{2n} and under p_3, C_{3n}

It is found for normally consolidated clays that c_u is directly proportional to p to a close approximation. This ratio can be expressed as

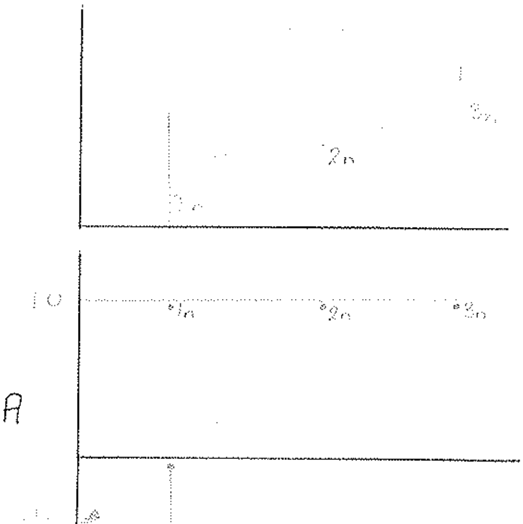
$$\left\{\frac{c_u}{p}\right\}_n = \text{constant}$$

If on the other hand the clay is consolidated under say p_2 and then allowed to swell the water content will be at ω_{1o} and the shear strength at c_{1o}.

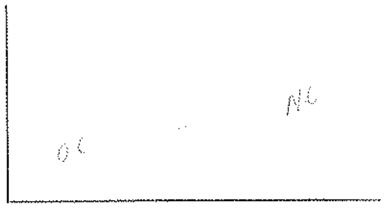
Thus samples I_n and I_o although under the same pressure have very different strengths

In order to determine the reason for the greater strength of the overconsolidated clay we must consider the A values.

In the N.C. cases $A_f = 1.0$ but in the O.C. cases $A_f \ll 1.0$ and hence the increase in strength of the O.C. samples is due to the pore pressure behaviour at failure and not the water content.



There is a change in the effective stress envelope as well but the loop is small ie the predominant influence on the strength is the A_f value.



The change in effective stress envelope is tied in with the w.c changes.

Change in undrained strength with depth.

At any particular consolidation pressure undrained tests show $\phi_u = 0$.

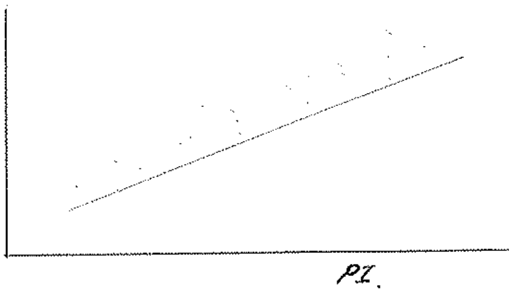
If the consolidation pressure is increased the C_u increases. If we have the same consolidation pressure but one sample is overconsolidated then for the reasons elaborated $Cu_{oc} > Cu_{NC}$. due to stress history affecting A_f.

However in all cases $\phi_u = 0$.

For normally consolidated samples there is a relation between $\left[\frac{C_u}{p}\right]_n$ and P.I.

The lower bound is given by

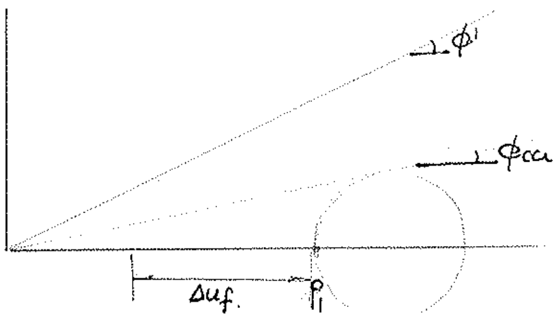
$$\left[\frac{C_u}{p}\right]_n = 0.11 + 0.0037[PI]$$



Consolidated undrained triaxial test.

Consolidate from a slurry two samples at p_1 & p_2 {N.C.} Then carry out an undrained test without a further change in cell pressure ($\Delta\sigma_3 = 0$) by increasing $\Delta\sigma_1$. The failure envelope w.r.t. total stresses gives the so called consolidated undrained angle of shearing resistance ϕ_{cu}.

NOTE: If the same was consolidated in the same way but failed by decreasing $\Delta\sigma_3$ then a totally different and much steeper line would be obtained.



In the standard test $\Delta\sigma_3 = 0$

$$\therefore \Delta u_{1f} = A(\Delta\sigma_1 - \Delta\sigma_3) = A_f\{\sigma_1 - \sigma_3\}_f.$$

$$\Delta u_2 = A_2 \{\sigma_1 - \sigma_3\}_f.$$

If both ϕ_{cu} and ϕ' envelopes are straight lines then ~~~~ $A_1 = A_2 = A$ at failure.
and hence is a constant for a normally consolidated soil

ie A is independent of consolidation pressure.

We have in general the relationship from the Mohr Coulomb failure criteria in terms of effective stress.

$$\left(\frac{\sigma_1 - \sigma_3}{2}\right)_f = \frac{c'\cos\phi' + (\sigma_3 - u)\sin\phi'}{1 - \sin\phi'}$$

In this case $\Delta u_f = A(\sigma_1 - \sigma_3)_f$
$c' = 0$

$$\therefore \quad \tfrac{1}{2}(\sigma_1 - \sigma_3)_f = \frac{\{\sigma_3 - A(\sigma_1 - \sigma_3)_f\}\sin\phi'}{1 - \sin\phi'}$$

$$\therefore \quad \tfrac{1}{2}(\sigma_1 - \sigma_3)_f(1 - \sin\phi') = p\sin\phi' - A\sin\phi'(\sigma_1 - \sigma_3)_f$$

$$\therefore \quad \tfrac{1}{2}(\sigma_1 - \sigma_3)_f\{1 - \sin\phi' + 2A\sin\phi'\} = p\sin\phi'$$

$$\therefore \quad \tfrac{1}{2}(\sigma_1 - \sigma_3)_f = \frac{p\sin\phi'}{1 + \sin\phi'(2A - 1)}.$$

$$\therefore \quad \left\{\frac{c_u}{p}\right\}_n = \frac{\sin\phi'}{1 + \sin\phi'(2A - 1)}$$

This equation also holds for sands but it must be noted that in that case A may be negative and $\phi_{cu} > \phi'$.

Using the conventional test it can be shown that.

$$\sin\phi_{cu} = \frac{\sin\phi'}{1 + 2A\sin\phi'}$$

$\sigma_1 - \sigma_3 = (\sigma_1 + \sigma_3)\sin\phi_{cu}$
$= (\sigma_1' + \sigma_3' + 2\Delta u)\sin\phi_{cu}$
$= \{\sigma_1' + \sigma_3' + 2A(\sigma_1 - \sigma_3)\}\sin\phi_{cu}$
$\therefore (\sigma_1 - \sigma_3)(1 - 2A\sin\phi_{cu}) = (\sigma_1' + \sigma_3')\sin\phi_{cu}$
$\therefore (1 - 2A\sin\phi_{cu}) = \frac{\sin\phi_{cu}}{\sin\phi'}$
etc.

If $A = 1.0$ and $\phi' = 30°$ then

$$\sin\phi_{cu} = \frac{1/2}{1 + 2A\sin\phi'} = 1/4$$

$$\text{ie} \quad \phi_{cu} \approx \tfrac{1}{2}\phi'$$

The use of this parameter ϕ_{cu} leads to the impression that from the undrained to the drained state one gets an increase in ϕ. This need not be the case and will not be so if A is -ve.

The strengths for $\left[\frac{C_u}{p}\right]$ measured in the undrained triaxial tests and vane tests on strata existing in nature in a N.C. state, when plotted against effective overburden pressure lead to a lower estimate of $\frac{C_u}{P}$ than is found with samples consolidated under equal all round stress in the laboratory. The difference increases as the PI decreases and hence the use of the results of consolidated undrained tests, expressed in terms of total stress either by the parameter $\frac{C_u}{P}$ or ϕ_{cu} can be justified in few practical applications.

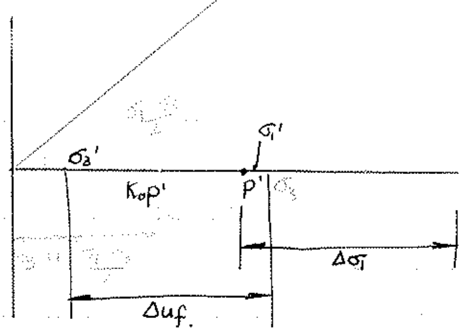
The reasons for the differences are.

1. A naturally deposited sediment is consolidated under conditions of no lateral strain and hence with a lateral effective stress considerably less than the vertical. The value of K_o is generally found to be from 0·35 – 0·7. The lower values tending to occur in soils of low PI. This cause alone can account for soils of low plasticity having a difference of 50% in the value of $\frac{C_u}{P}$.

2/ Reconsolidation in the lab after the stress release associated with even the most careful sampling leads to a low void ratio than would actually occur in the insitu material. The value of A ~~is~~ in particular is sensitive to this modification in structure and this leads to a higher undrained strength.

Tests on samples anisotropically consolidated in the lab and on undisturbed samples show that the effective stress ~~stren~~ in the sample when under an all round pressure or unconfined < ½ the effective overburden pressure in situ (ie before any deviator stress applied). Yet when failed the strengths agree well with field & vane tests. This is probably due to the fact that strength uniquely related to water content for limited range of soils & stress paths. Hence to reconsolidate under the existing overburden pressure will give too high a value of $\frac{c_u}{p}$.

Analysis of effect of anisotropic consolidation



An element of clay in the ground is initially subjected to a deviator stress $p\{1-K\}$

Pore pressures are set up by the additional deviator stresses required to fail the sample under undrained conditions

If the principal stress directions remain the same during consolidation and subsequent failure and if no initial excess pore pressures exist after consolidation then the Mohrs circles can be drawn as shown and $\Delta\sigma_1$ and $\Delta\sigma_3$ are the total undrained stress changes

σ_3 = All round pressure.
$\neq p$ as in 1st case

$$\therefore \Delta u_f = \Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)$$
$$= \{\sigma_3 - K_0 p\} + A\{\sigma_1 - p - \sigma_3 + K_0 p\}$$
$$= \sigma_3 - K_0 p + A(\sigma_1 - \sigma_3 - (1-K_0)p\}$$

From the geometry of the Mohrs circle.

$$\frac{\sigma_1-\sigma_3}{2} = \sin\phi' \left\{\sigma_3 - u + \frac{\sigma_1-\sigma_3}{2}\right\}$$

Thus
$$\frac{\sigma_1-\sigma_3}{2} = \sin\phi'\left\{\sigma_3 - \left[\sigma_3 - K_0 p + A(\sigma_1-\sigma_3-(1-K_0)p)\right] + \frac{\sigma_1-\sigma_3}{2}\right\}$$

$$\therefore \frac{\sigma_1-\sigma_3}{2}\left\{1 - \sin\phi' - 2A\sin\phi'\right\} = \sin\phi'\left[p\{(1-K_0)A + K_0\}\right]$$

$$\therefore \frac{\sigma_1-\sigma_3}{2} = \frac{p\sin\phi'\left[A(1-K_0)+K_0\right]}{1 + (2A-1)\sin\phi'}$$

If $K_0 = 1.0$ get same expression as before.

a lab test in which samples are consolidated under an equal all round pressure will give values of $\frac{c_u}{p}$ that are too high. ie an overestimate of strength.

In actual fact A_f values are usually greater in the anisotropically consolidated case than under all round pressure.

This has the effect of making the anisotropically consolidated values even lower.

This form of theoretical ~~work~~ will not work in the passive case because the stress directions change. It is necessary in that case to do the applicable test.

Effective stress parameters

If pore pressures are measured then c' and ϕ' obtained. These values are usually based on maximum deviator stress. $(\sigma_1' - \sigma_3')_{max}$.

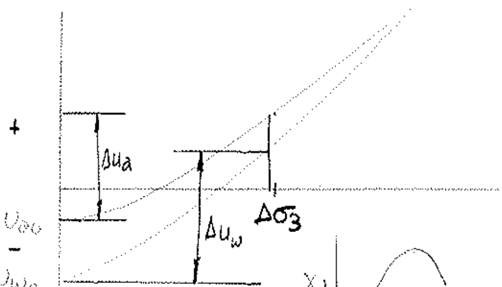
In some overconsolidated clays where large failure strains are found due to decrease in pore pressure on shear a larger ϕ' value is obtained by plotting σ_1'/σ_3', and the max value may occur at fairly small strains

In comparing ϕ_d' and ϕ_{cu}' it is necessary to compare the definition being used.

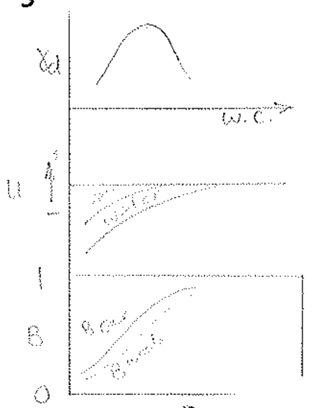
Undrained Test on Partly saturated soil

The pore pressure changes on the application of an all round pressure $\Delta\sigma_3$ have been considered earlier.

$$\Delta u_a = B_a \, \Delta\sigma_3$$
$$\Delta u_w = B_w \, \Delta\sigma_3$$
and
$$u_a = u_{a0} + \Delta u_a$$
$$u_w = u_{w0} + \Delta u_w$$



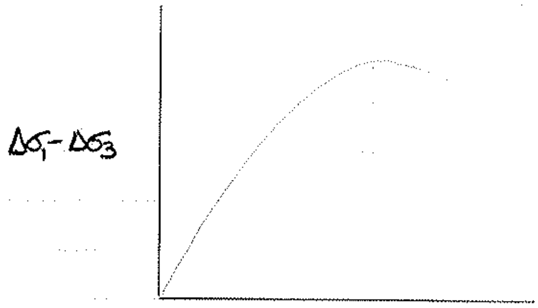
Now $u_a - u_w$ = +ve due to capilarity and varies with the compaction conditions in samples of compacted fill.



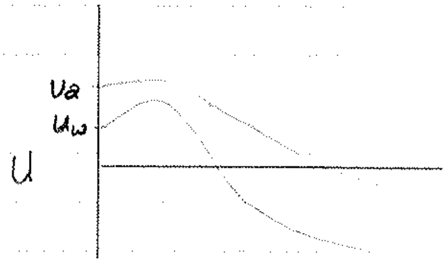
Pore pressure changes under the deviator stress.

ie have already applied $\Delta\sigma_3$ and so the pressures are u_a and u_w

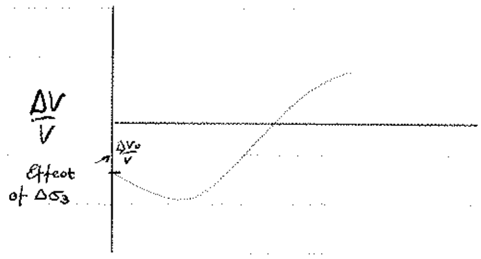
Although the sample is undrained there is a volume change which shows an initial decrease in volume followed by dilation



As the volume decreases initially the $u_a - u_w$ difference becomes smaller because the bubbles become larger



We thus have in general



$$\Delta u_a = B_a\left[\Delta\sigma_3 + A_a(\Delta\sigma_1 - \Delta\sigma_3)\right]$$

$$\Delta u_w = B_w\left[\Delta\sigma_3 + A_w(\Delta\sigma_1 - \Delta\sigma_3)\right]$$

These factors do not however have much predictive value.

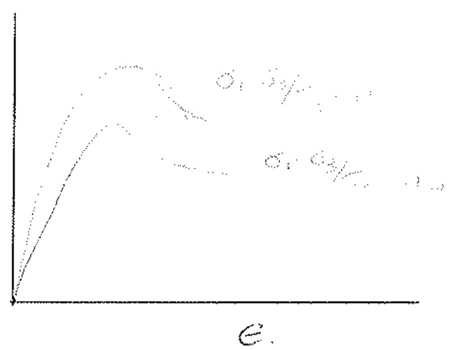
It is useful to plot the results in terms of principal stress ratio $\frac{\Delta\sigma_1'}{\Delta\sigma_3'}$

Now since $\Delta\sigma_3$ is applied initially and then σ_1 is increased $\sigma_1 - \sigma_3 = \Delta\sigma_1'$ since σ_3 is constant.

$\sigma_3 - u_w = \Delta\sigma_3'$ w.r.t water.

$\sigma_3 - u_a = \Delta\sigma_3'$ w.r.t air.

Principle Stress Ratio



Effects of time on the measurements.

a/ Change in pore pressure under constant all round stress

- a: Tendency for u_w to decrease with time after remoulding
- b: Diffusion of air through membrane

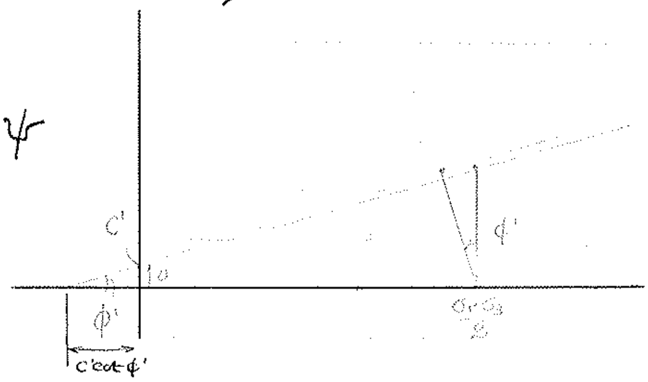
b/ Time lag of equilization throughout sample during shear.

A probe was used to measure u_w at the shear plane at the same time as measurements were made at the base. It was shown that a very slow time was required for $u_{w\,probe} = u_{wb}$. Otherwise the pore pressure at the probe was lower than at the base. (dilation on shearing).

Using the u_{wb} are valid (when testing too quickly) get a much larger c' than using $u_{w\,probe}$.

Determination of parameter χ

Plot used in order to avoid the use of too many Mohr's circles. The diameter of the circles is plotted



$$\frac{\sigma_1 - \sigma_3/2}{\sigma_1 + \sigma_3/2 - u + c'\cot\phi'} = \sin\phi' = \tan\psi$$

ie $\underline{\sin\phi' = \tan\psi}$

and $\dfrac{d}{c'\cot\phi} = \tan\psi$

Thus $\underline{c' = d}\ \dfrac{\tan\phi'}{\tan\psi}$

Thus a graph of $\dfrac{\sigma_1 - \sigma_3}{2}$ vs $\dfrac{\sigma_1 + \sigma_3}{2} - u$ can be plotted and any line can easily be expressed in the true c' and ϕ' values.

To consider the effects of partial saturation on the shear strength we have the equation

$$\sigma' = \sigma - u_a + \chi(u_a - u_w).$$

The value of $\chi = 1{\cdot}0$ saturated soils and $\chi = 0$ dry soils.

Types of test used

1. Undrained test

measure pore water & pore air pressure.

- problems due to cavitation
- " " " diffusion of air through rubber.

2. Constant water content test.

The air pressure in the pore space is controlled independently - allowed to equal atmospheric or else boosted to raise u_w to the range where it can be measured.

3. Consolidated undrained with full saturation

Cell pressure increased and back pressure on water increased until saturation reached.

After saturation the sample is sheared with no. ~~full~~ drainage with p.w.p measured.

4. Drained test.

Same procedure as above but full drainage during shear.

Measurement of water pressure.

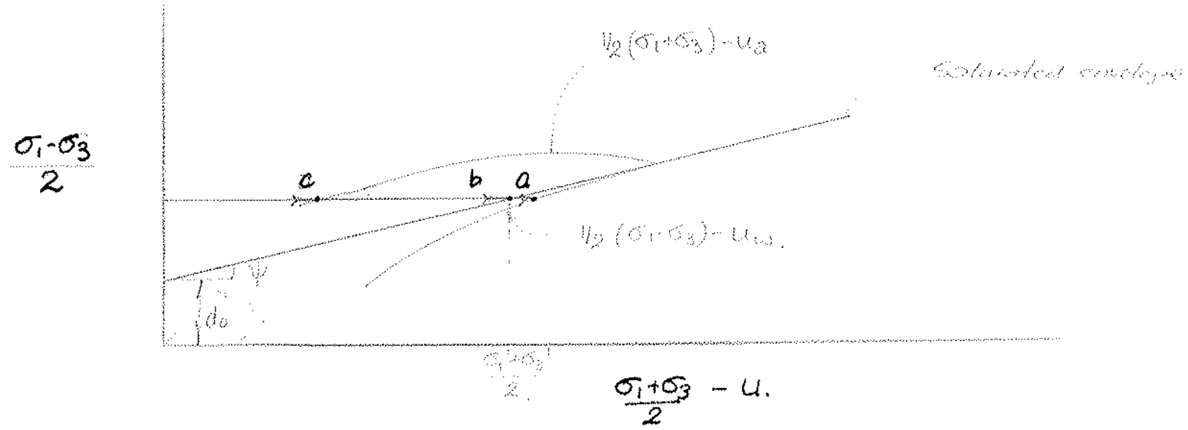
A ceramic disc of high air entry value is necessary. especially in the constant water content tests where the air pressure is raised to enable the water pressure to be measured.

Aerox "Celloton" air entry value $= 30$ psi

$k = 2{\cdot}9 \times 10^{-6}$.

In practical work one must use either the saturated parameters or the parameters obtained when u_w is measured. Measuring u_a will result in an overconservative design.

The tests usually performed are either <u>undrained</u> or <u>constant water content tests</u>. Both u_w and u_a are measured and it is assumed that ϕ' from fully saturated soils is unique.



<u>NOTE:</u> 1. The smallest value of $\frac{\sigma_1+\sigma_3}{2} - u_a$ is when

$u_a = \sigma_3$ thus $\frac{\sigma_1+\sigma_3}{2} - u_a = \frac{\sigma_1-\sigma_3}{2}$

Thus cannot get test values below the 45° line

2. If one extends the envelopes obtained using u_a and u_w back there will be different cohesion intercepts depending on what one is measuring ie u_a or u_w. Hence the reason why the use of a coarse porous stone leads to large cohesion intercepts.

<u>Determination of χ</u>

$$\sigma' = \sigma - u_a + \chi(u_a - u_w)$$

Thus $$\tfrac{1}{2}(\sigma_1'+\sigma_3') = \tfrac{1}{2}(\sigma_1+\sigma_3) - u_a + \chi(u_a - u_w).$$

From the saturated sample effective stress envelope we let

$$b = \frac{\sigma_1'+\sigma_3'}{2}$$

also

$$c = \frac{\sigma_1+\sigma_3}{2} - u_a.$$

and $$u_a - u_w = a - c.$$

Thus $$\chi = \frac{b-c}{a-c}.$$

Assumptions

1/ The structure of the material remains the same ie the true envelope is not affected by the degree of saturation.

2/ The fact that if one extends the plot of $\frac{\sigma_1+\sigma_3}{2} - u_w$ vs $\frac{\sigma_1-\sigma_3}{2}$ back one gets a negative

c' (d) value indicates the need for a more complicated effective stress envelope.

Notes on other types of test.

a. Consolidated undrained.

The sample is first saturated using a back pressure. This is done in stages and after each stage an increment of $\Delta\sigma_3 = 10$ psi is used to check if $B = 1$. It is possible that saturation may occur at a lower back pressure than predicted by the earlier theory because of diffusion of air through the membrane.

The undrained test is then carried out at a speed appropriate to measuring the pore pressure at the base of the sample.

b. Drained test.

Saturate as above and then shear as for a saturated soil.

Note: The undrained C_u and ϕ_u values depend radically on the degree of saturation but $\phi_u \to 0$ at high enough stress levels. This may be useful only in a very fatty clay.

Use of field measurements.

If possible use a piezometer that measures u_w.

For practical work use either saturated parameters or those determined by measuring u_w.

Effects of volume change at failure.

The rate of volume change at failure is found to vary with degree of saturation for even the same stress history. This influences the relevant value of ϕ' (ie a drained test and an undrained test give slightly different values of ϕ') Can be corrected.

The conventional drained test with σ_3' constant has coincidence between max principal stress ratio and max deviator stress

$$\text{ie} \quad \frac{\sigma_1 - \sigma_3}{\sigma_3'} \quad \& \quad (\sigma_1 - \sigma_3)_{max}.$$

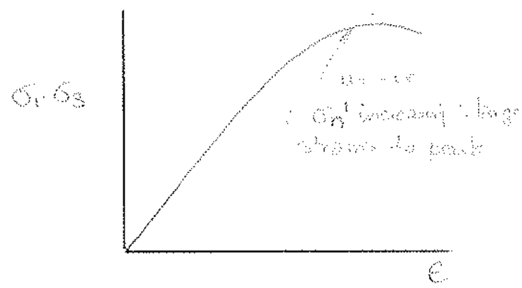
There is however a volume change at failure. There is a good correlation between drained tests, with an energy term subtracted due to dilation at failure. and undrained test results at peak principal ~~deviator~~ stress. ratio

$$\frac{\Delta\sigma_1'}{\Delta\sigma_3'} = \frac{\sigma_1 - \sigma_3}{\sigma_3 - u}$$

For problems of long term stability the strength including the energy term is required so the value from the undrained test from $\frac{\sigma_1 - \sigma_3}{\sigma_3'}$ is conservative.

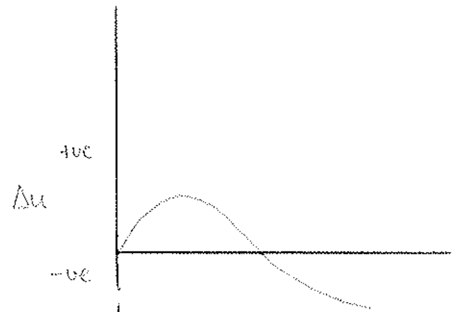
Peak Deviator stress - Maximum principal stress ratio

<u>Consider the failure of an overconsolidated soil or a saturated medium dense sand. Undrained.</u>

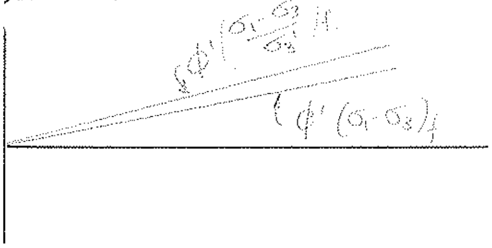


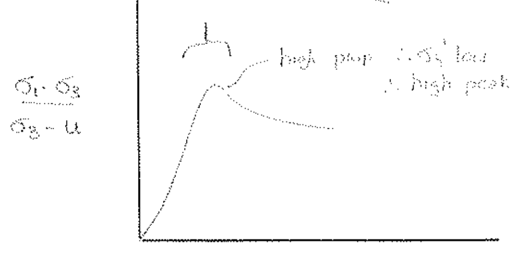
Plotting maximum stress ratio will give a different failure envelope from the normal max. deviator stress plot.

The problem is which result to use.



It seems that if there is to be no p.w.p. dissipation before failure then the lower value $(\sigma_1 - \sigma_3)$ should be used.

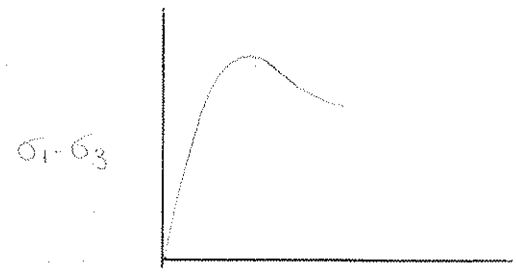




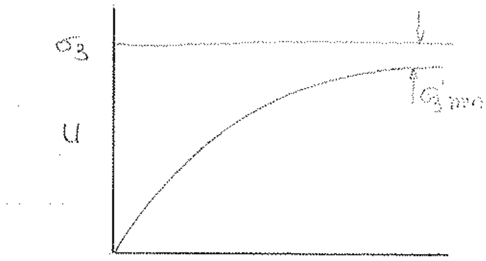
If there will be dissipation of pore water pressure then the higher value (max stress ratio) can be used. This gives values closer to the drained test results.

Loose sands or quick clays.

Peak Deviator

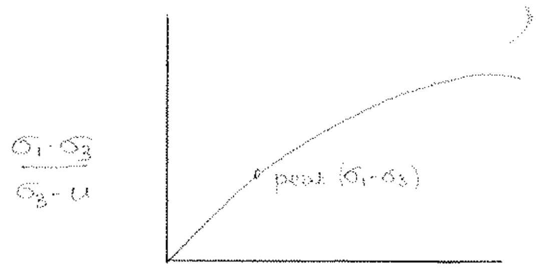


Because of the large positive pore pressures set up under shear the value of σ'_{nf} on the failure plane is reached very soon. ie the general equation



$$\left(\frac{\sigma_1 - \sigma_3}{2}\right)_f = \frac{(\sigma_3 - u)\sin\phi'}{1 - \sin\phi'}$$

shows clearly that if Δu_f is large and positive the value of $(\sigma_1 - \sigma_3)_f$ will be smaller than if u is negative say.

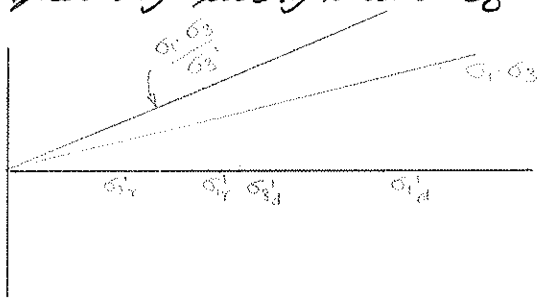


Maximum stress ratio

$$\frac{\sigma_1 - \sigma_3}{\sigma_3 - u} = \frac{2\sin\phi'}{1 - \sin\phi'}$$

If u is becoming larger with increasing strain then the peak value of $\frac{\sigma_1 - \sigma_3}{\sigma_3 - u}$ will continue to increase until maximum 'u' is reached

Now it is an experimental fact that plotting the Mohr envelope using max. deviator stress does not give the same value of ϕ' as plotting using values of maximum stress ratio



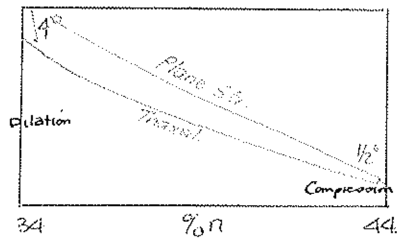
In some cases ϕ' from max deviator stress may be ½ from $\frac{\Delta\sigma_1'}{\Delta\sigma_3'}$.

One should not use the value obtained from $\frac{\Delta\sigma_1'}{\Delta\sigma_3'}$ readings if performing usual effective stress calcs. The cause of the difference is the high build up of positive pore pressure in loose granular material.

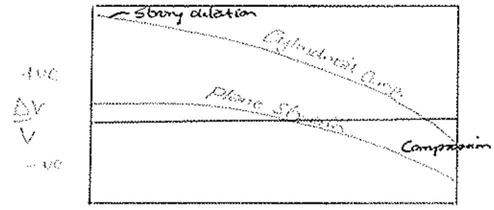
The Effect of the Intermediate principal stress.

Plane strain tests:

$$\sigma_2' \approx 0.6 \frac{(\sigma_1 + \sigma_3)}{2}$$

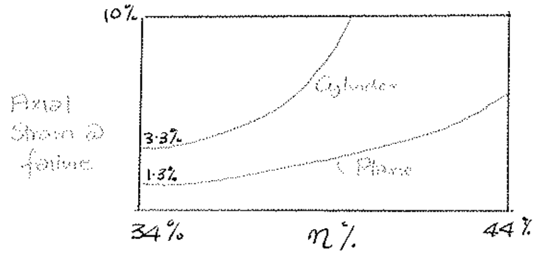


Tests on sand: Results shown in figs 1-3.



Differences:

a. Plane strain gives sharper peak at a higher strength. but both give approximately the same residual strength
ie at a given porosity material under plane strain is more brittle.



b. Smaller dilation under plane strain. The material shears at constant volume whereas in the triaxial the dilation continues for much larger strains before flattening off. For loose material the differences almost disappear.

c. The standard triaxial test tends to underestimate the result. It appears that the difference is due to the different volume changes under shear. If the stresses are large enough to suppress the rate of change of volume during shear then the difference between the two strength values may be zero.

Tests on compacted moraine.

Considering $\sigma_1 - \sigma_3 / \sigma_3'$ max the difference in $\phi' = 4.7°$
Considering $\sigma_1 - \sigma_3$ max " " " $\phi' = 1.8°$

The problem is that the Mohr - Coulomb failure criteria does not take account of σ_2' and it is necessary to investigate as to whether it is correct. ie to what extent is it wrong.

Failure criteria for cohesionless soils

Two requirements of a failure criteria

1/ Strength should be proportional to the normal stress for a given stress system.

2/ The influence of the intermediate principal stress should be correctly indicated

Failure criteria which obey at least the first requirement.

Mohr Coulomb

$$\tau = \sigma_n' \tan \phi'$$

$$\frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} = \sin \phi'$$

Extended Tresca.

$$\sigma_1' - \sigma_3' = \alpha \frac{(\sigma_1' + \sigma_2' + \sigma_3')}{3}$$

Extended von Mises

$$(\sigma_1' - \sigma_2')^2 + (\sigma_2' - \sigma_3')^2 + (\sigma_3' - \sigma_1')^2 = 2\alpha^2 \frac{(\sigma_1' + \sigma_2' + \sigma_3')^2}{3}$$

Extension & Compression

The Mohr Coulomb criteria indicates that σ_2' has no influence on the strength. Hence the same principal stress ratio at failure would be expected for both extension and compression.

However both Tresca & Von Mises criteria show important differences between the stress ratio in compression and in extension

In compression $\sigma_2' = \sigma_3'$

Both criteria reduce to $\sigma_1' - \sigma_3' = \alpha \frac{(\sigma_1' + 2\sigma_3')}{3}$

In Extension $\sigma_2' = \sigma_1'$

Both criteria reduce to $\sigma_1' - \sigma_3' = \alpha \frac{(2\sigma_1' + \sigma_3')}{3}$.

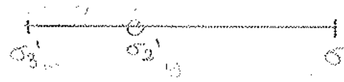
The influence of σ_2' on the strength is most readily seen by plotting variations of ϕ' as σ_2' varies between the limits of σ_3' and σ_1'.

The Mohr Coulomb criteria may be written

$$\frac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} = \sin \phi'$$

If we let b indicate the relative position of σ_2' between σ_3' and σ_1'

ie $b = \dfrac{\sigma_2' - \sigma_3'}{\sigma_1' - \sigma_3'}$



Then we can write the other criteria as follows.

8: Extended Tresca $\quad \dfrac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} = \dfrac{1}{1/3 + 2/\alpha - 2/3\, b} \quad \left\{ = \sin\phi' \text{ by def}^n \right\}$

9: Extended von Mises $\quad \dfrac{\sigma_1' - \sigma_3'}{\sigma_1' + \sigma_3'} = \dfrac{1}{1/3 + 2/\alpha\sqrt{1 - b + b^2} - 2/3\, b}$

Hence ϕ' predicted by equations 8 & 9. varies between.

$$\sin^{-1} \frac{1}{1/3 + 2/\alpha} \quad \text{and} \quad \frac{1}{2/\alpha - 1/3}.$$

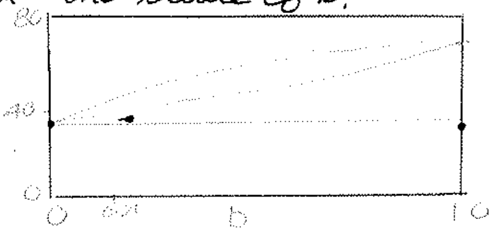
Since α at failure is usually > 0.8 this means a large % difference in the values of ϕ' at failure.

$$\alpha = \frac{\sigma_1' - \sigma_3'}{1/3\,(\sigma_1' + \sigma_2' + \sigma_3')}$$

The main point of interest is the variation of ϕ' as defined by Mohr Coulomb with the value of b.

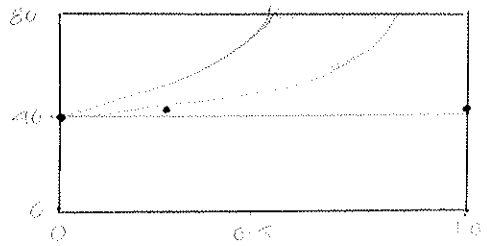
Data for loose sand $\quad \alpha = 1.375$

$\phi_c' = 34\%$



Data for dense sand $\quad \alpha = 1.636$

$\phi_c' = 40°$.



For both loose and dense sands the Mohr Coulomb criteria gives the best overall fit

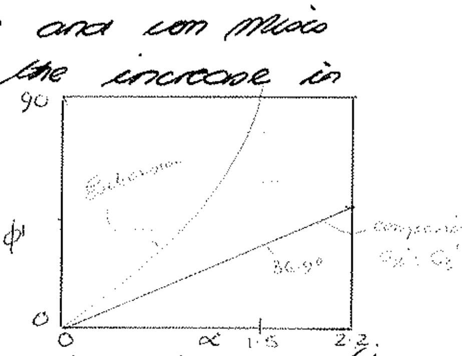
For the dense sand the Tresca gives a slightly better fit when considering compression and plane strain

However for the extension tests the extended Tresca and the extended von Mises both fail to predict meaningful results

The reason for this becomes apparent when considering the following.

The parameter α in both the Tresca and von Mises failure criteria is used to indicate the increase in strength with normal stress.

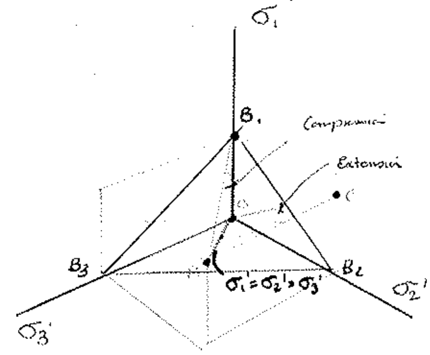
If the values of ϕ' in compression predicted by the different criteria are plotted against the value of α then the following becomes apparant.



For the compression test the relationship between ϕ' and α is almost linear and approximates to $\phi' = 25\alpha$ over the range 20° - 40°. However the value predicted in extension rapidly diverges from that in compression and $\phi'_{ext} = 90°$ @ $\alpha = 1.5$ at which value $\phi'_c = 36.9°$

The reason for this is simply that for values of $\phi' > 36.9°$ the von Mises and Tresca criteria predict results in the negative stress space - which for soils is quite in-admissable.

<u>Three dimensional stress space.</u>



<u>Triaxial test.</u>

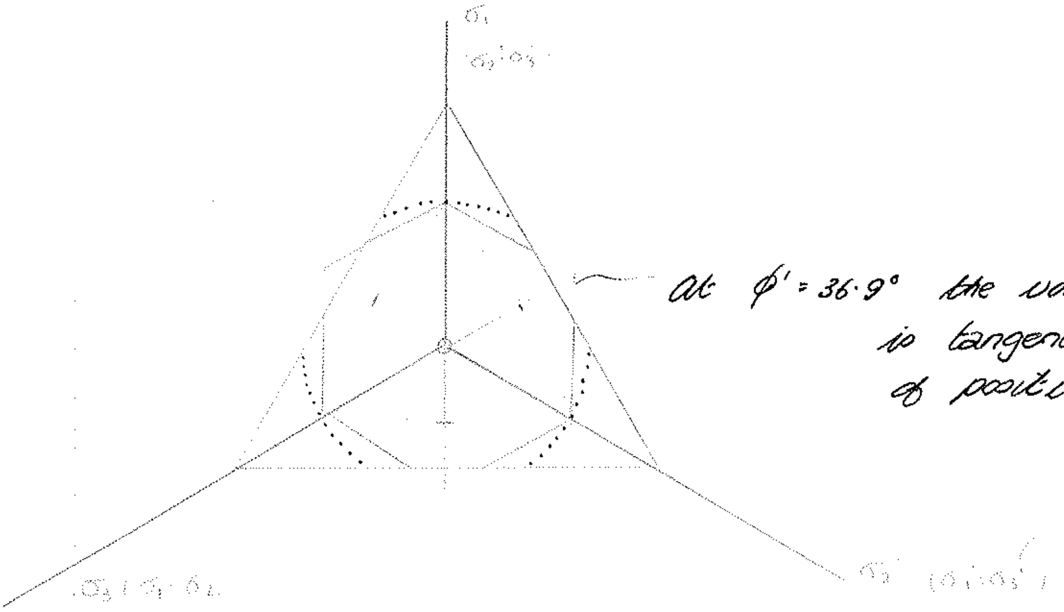
a. <u>Compression</u> $\sigma_1' > \sigma_2' = \sigma_3'$

b. <u>Extension</u> $\sigma_1' = \sigma_2' > \sigma_3'$

In general the stress condition of a soil can only be represented by a point somewhere in the volume OABC. because $\sigma_1', \sigma_2', \sigma_3'$ must always all be positive and $\sigma_1' \geqslant \sigma_2' \geqslant \sigma_3'$

States of limiting equilibrium expressed by the various failure criteria are represented by pyramid shaped surfaces having their apex at O and showing characteristic sections in the plane $B_1 B_2 B_3$ (which is normal to the line $\sigma_1' = \sigma_2' = \sigma_3'$

These characteristic sections are shown for values of ϕ' less and then more than 36.9°. It can be quite clearly seen that as σ_2' moves from σ_3' to σ_1' the extended von Mises and Tresca criteria, for $\phi' > 36.9°$ move out into negative stress space



at $\phi' = 36.9°$ the von Mises circle is tangential to the envelope of positive stresses.